The Economic Value of Commodities in Asset Allocation when Returns are Predictable

John Cotter, Emmanuel Eyiah-Donkor,* and Valerio Potì
Department of Banking and Finance, Michael Smurfit Graduate Business School, University College Dublin, Ireland

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Abstract

We evaluate the out-of-sample performance of commodities in portfolios composed of stocks, bonds, and T-bills. The evidence on their ability to generate economic value is mixed, with previous studies ignoring the potential role of asset return predictability, and the states of the business cycle. Using monthly data over the period 1976-2015, we document sizeable utility gains for a mean-variance investor who exploits predictability in asset returns. For example, an investor with moderate level of risk aversion, who imposes sensible restrictions on portfolio weights, can generate net-of-transactions-costs utility gains of over 130 basis points per annum. In addition, we find strong influence of the business cycle, as dated by the National Bureau of Economic Research, on portfolio performance. During the recessionary phase of the business cycle, commodities are shown to generate substantial utility gains of over 1338 basis points per annum. In expansionary periods of the business cycle, however, commodities do not add economic value generating utility losses of over 50 basis points. These findings suggest that the ability of commodities to improve portfolio performance is countercyclical. Our findings are robust to varying levels of risk aversion, portfolio weight constraints, transactions costs and alternative performance metrics.

JEL classification: C10, C32, C53, G11

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1. Introduction

Is there a role for commodities in portfolios composed of stocks, bonds, and T-bills? This is an important question given the popular investment advice, as established by portfolio theory, that investors should hold portfolios that diversify across different asset classes so as to improve portfolio performance, and the inconclusive evidence on their ability to generate sizeable economic gains reported thus far in the literature (see, for example, Bessler and Wolff (2015); You and Daigler (2013); Daskalaki and Skiadopoulos (2011)). Our set-up naturally allows us to consider two types of investors with and without commodities whose portfolio performance we compare out-of-sample using a set of standard performance metrics. Both investors, with mean-variance preferences, follow dynamic strategies and can choose between the four asset classes. The first investor's portfolio is composed of traditional asset classes: stocks, bonds and

*Corresponding author: Emmanuel Eyiah-Donkor, emmanuel.eyiah-donkor@ucdconnect.ie, Tel: +353-89-9801301; John Cotter, john.cotter@ucd.ie; Valerio Potì, valerio.poti@ucd.ie. The authors would like to acknowledge financial support of Science Foundation Ireland under Grant Number 08/SRC/FM1389. Emmanuel Eyiah-Donkor also gratefully acknowledges support of a generous UCD Business School Doctoral Scholarship.

risk-free bills (traditional portfolio), whereas the second investor's portfolio includes all the four asset classes (commodity portfolio).

To examine this question, we contribute to the literature by extending previous studies in three ways. First, we evaluate the ability of commodity futures to improve the out-of-sample performance of a portfolio composed of stocks, bonds, T-bills by assuming that returns are predictable.[1] There is surprisingly little empirical work on the economic value of commodities in asset allocation in the presence of time-varying expected returns. Such predictability for the stock market has been documented in, for example, Campbell and Thompson (2008), Pettenuzzo, Timmermann, and Valkanov (2014), Johannes, Korteweg, and Polson (2014), to have implications for optimal portfolio choice.[2] We argue that the inconclusive evidence documented in previous studies could in part be due to the nature of their research design which focusses on in-sample evidence, and the use of constant expected returns in the asset allocation exercise thus ignoring the important feature of time-varying expected returns. To the best of our knowledge, Giamouridis, Sakkas, and Tessaromatis (Working paper, 2014) is the first to make an attempt on this problem. Working in the VAR framework of Campbell, Chan, and Viceira (2003), they find no economic value to investing in commodities.

Second, we examine the potential influence of the state of the business cycle as dated by NBER on the performance of commodities and their role in the asset allocation. Recent evidence by Rapach et al. (2013) and Henkel, Martin, and Nardari (2011), respectively, Gargano and Timmermann (2014) for stocks and commodity spots find that predictability is stronger during recessionary states of the economy relative to expansions. The economic rationale for including commodities in portfolios is their ability to act as an inflation hedge. Indeed, as inflation rate depends on the prices of physical commodities, commodity futures could very well be used an a proxy for inflation. Also, considering that inflation expectations are important for formulating monetary policy, the state of the business cycle may very well be related to the performance of commodity futures and their performance thereof in portfolios. The inconclusive evidence documented in previous studies may thus be the result of ignoring this important feature.

Our third contribution extend prior research by studying a much longer sample period from spanning 40 years (1976-2015). By using a longer sample, we avoid the potential pitfall of periods of good and bad performance contaminating our results. The longer sample should also enables us to have sufficient samples to undertake separate analysis for states of the business cycle.

The dataset for our empirical analysis consist of monthly returns on stocks, bonds, T-bills, and commodity futures, as well as economic variables that have been used in the predictability literature for decades described in Section 3. Our in-sample and out-of-sample tests of predictive ability for the potential predictors for the various asset classes indicates that while the individual economic variables shows statistically insignificant predictive ability at conventional levels, the combination forecasts deliver out-of-sample R^2 statistic that are statistically signi-

[1] We consider commodity futures rather than the physical commodity because of the relative ease in buying and selling commodity futures contracts. Unlike stocks and bonds, a direct investment in the physical commodities whiles generally unrealstic, is often characterized by high transactions and storage costs.

[2] See, for example, Welch and Goyal (2008), Cochrane (2008), Campbell and Thompson (2008), and Rapach, Zhou, et al. (2013) for a review.

ficant. Predictability is much stronger for recessions relative to expansions when examined for NBER-dated business cycle expansions and recessions.

In portfolio performance analysis using the Sharpe ratio, certainty equivalent return gain, and portfolio turnover indicate substantial economic value of commodities in asset allocation to a mean-variance investor who, based on the evidence of return predictability from macroeconomic and financial variables, combine forecasts from individual bivariate predictive regressions. Our results also show a the role for the business-cycle. Specifically, we find net-of-transactions-costs certainty equivalent gains during recessions relative to expansions for the full out-of-sample and second out-of-sample periods, and expansions relative to recessions for the first out-of-sample period. The profitability of investing in commodities is concentrated during the recessionary phase of the business cycle. These results, however, are subject to the use of moderate investor risk aversion and imposing sensible constraints on portfolio weights. Overall, our results reverse the conclusions in Daskalaki and Skiadopoulos (2011) and to some extent Bessler and Wolff (2015) who assume a constant expected returns model in their asset allocation exercise and ignore the state of the business cycle.

Commodities as an asset class has been suggested for inclusion in investors portfolio because of their historically low correlation with traditional asset classes, with Sharpe ratios almost comparable to that of stocks (see, for example, Gorton and Rouwenhorst (2006)). Modern portfolio theory suggests that combining less correlated assets should lead to portfolio risk diversification and therefore economic gains. However, the evidence on the economic value of commodities in asset allocation documented in previous studies have so far been inconclusive. Jensen, Johnson, and Mercer (2000) investigate whether the efficient frontier shifts when commodities, proxied by the S&P GSCI commodity index futures, are incorporated into a portfolio consisting of stocks, bonds, T-bills and real estate and finds diversification benefits. Belousova and Dorfleitner (2012) document similar results by conducting extensive statistical tests, while You and Daigler (2013) document out-of-sample benefits using individual commodity futures returns.

Recent works by Daskalaki and Skiadopoulos (2011), and Giamouridis et al. (Working paper, 2014) find evidence to the contrary. There is also another strand of literature, including Domanski and Heath (2007) and Tang and Xiong (2012), that cast doubt on the diversification potential of commodities by arguing that the financialization of the commodity markets has increased their correlation with stocks and bonds. The first obvious limitation of previous studies is the widespread focus on in-sample analysis (although there are few studies such as, Daskalaki and Skiadopoulos (2011) and Giamouridis et al. (Working paper, 2014), that provide out-of-sample evidence).[3] Bessler and Wolff (2015) implements a number of portfolio strategies and finds that although commodities add economic value to a stock-bond portfolio for some of the implemented portfolio strategies, the attainable gains are much smaller when compared to the evidence documented in in-sample analysis. They shows that the results is concentrated for popular commodity indices and energy commodities but not for agriculture and livestock. Their studies is also limited in the sense that they do not perform any tests to check whether the economic gains are statistically significant.

[3] Daskalaki and Skiadopoulos, for example, consider an investor with preferences for skewness and kurtosis of the return distribution and finds that, in-sample, commodities provide added value but not out-of-sample.

The rest of the paper is organised as follows. Section 2 develops our methodology for measuring the economic value of commodities. Section 3 describes the data used in our paper and offers preliminary analysis. Section 4 analyses the empirical results for the asset allocation exercise. Section 5 concludes.

2. Methodology

Our methodology for assessing the economic value of commodities in asset allocation is to evaluate the performance of a portfolio that include commodities against a portfolio that ignores commodities when asset returns are predictable. To develop our methodology, let $\boldsymbol{r}_{t+1} = \boldsymbol{R}_{t+1} - \boldsymbol{\iota} R_{f,t+1}$ denote the N-vector of risky assets excess returns, where $\boldsymbol{R}_{t+1}$ is the N-vector of risky asset returns, $R_{f,t+1}$ is the risk-free rate of return, and $\boldsymbol{\iota}$ denotes an N-vector of ones.

2.1. Asset Allocation Exercise

Consider a utility maximizing investor who allocates her wealth between four asset classes: stocks, bonds, commodities and risk-free bills. Our investor is assumed to have access to information on economic variables that she thinks can forecast the future payoff of asset returns, and therefore she is able to follow dynamic portfolios by timing expected returns using predictive regression forecasts of stock, bond and commodity excess returns. For a given level of initial wealth, which we normalize to 1, the investor chooses portfolio weights $\boldsymbol{w}_t$ on the N-vector of risky assets so as to maximize the following mean-variance objective function:

$$U(R_{p,t+1}) = \mathrm{E}_t(R_{p,t+1}) - \frac{\gamma}{2}\mathrm{Var}_t(R_{p,t+1}), \tag{1}$$

where $R_{p,t+1}$ is the portfolio returns, and γ is the coefficient of relative risk aversion. At the end of month t, the investor optimally allocates the following share of her portfolio to the risky assets during the subsequent month $t+1$:

$$\boldsymbol{w_t} = \frac{1}{\gamma}\boldsymbol{\Sigma}_{t+1}^{-1}\boldsymbol{\mu}_{t+1}, \tag{2}$$

where $\boldsymbol{\mu}_{t+1} = \mathrm{E}_t[\boldsymbol{r}_{t+1}]$ is the N-vector of simple excess return forecasts[4] and $\boldsymbol{\Sigma}_{t+1} = \mathrm{E}_t[(\boldsymbol{r}_{t+1} - \boldsymbol{\mu}_{t+1})(\boldsymbol{r}_{t+1} - \boldsymbol{\mu}_{t+1})']$ is the $N \times N$ covariance matrix forecast of the N risky assets returns. The share of wealth allocated to the risk-free bills is $1 - \boldsymbol{\iota}'\boldsymbol{w}_t$, and the month $t+1$ portfolio returns is given by

$$R_{p,t+1} = R_{f,t+1} + \boldsymbol{w}_t'\boldsymbol{r}_{t+1}. \tag{3}$$

2.2. Modelling Conditional Moments of Excess Returns

In order to compute the optimal portfolio weights, $\boldsymbol{w}_t$, defined in Equation (2), we need the one-step ahead forecasts of the vector of conditional means, $\boldsymbol{\mu}_{t+1}$, and conditional variance-covariance matrix $\boldsymbol{\Sigma}_{t+1}$ of portfolio excess returns.

[4] We implement the portfolio strategy using simple (instead of log) returns so that the resulting portfolio returns are given by the sum of the product of portfolio weights and asset excess returns.

Consider the following bivariate predictive regression model

$$r_{i,t} = \alpha_i + \beta_i x_{j,t-1} + \varepsilon_{i,t}, \quad i = 1, ..., N, j = 1, ..., K \tag{4}$$

where $r_{i,t}$ is the simple excess return on the i^{th} asset from time $t-1$ to t, $x_{j,t}$ is a potential predictor of asset i available at time $t-1$, and $\varepsilon_{i,t}$ is a zero-mean error term. The predictor variables we consider, and are selected based on evidence documented in prior studies of their predictive ability for the asset classes, are defined in 3.1.

Suppose T observations are available for $r_{i,t}$ and $x_{j,t}$. To initialize our parameter estimates, we use $n_1 = 167$ observations (1976:02-1989:12) as the in-sample estimation period, and the remaining $T - n_1 = 312$ observations as the out-of-sample evaluation period (1990:0-2015:12). The parameters are updated recursively as new data becomes available, meaning that the estimation sample always starts in 1976:02 expanding the estimation window by one as additional observations become available. Only data up to the previous month is therefore used to estimate the model parameters and generate the pseudo out-of-sample forecast of excess returns corresponding to each predictor variable for the current month, $t+1$, as

$$\mathrm{E}_t[r_{i,t+1}] = \mu_{i,t+1} = \hat{\alpha}_i + \hat{\beta}_i x_{i,t}, \tag{5}$$

where $\hat{\alpha}_i$ and $\hat{\beta}_i$ are the OLS estimates of α_i and β_i in equation (4), respectively, from regressing $\{r_{i,s}\}_{s=2}^{t}$ on a constant and $\{x_{j,s}\}_{s=1}^{t-1}$.

In response to the poor performance of forecasts based on individual predictor variables (see, for example, Welch and Goyal (2008)), several combination forecast approaches have been developed since the seminal work of Bates and Granger (1969) to address these issues to reduce parameter estimation error and improve forecasts. Furthermore, because it is difficult to know a priori which economic variables contain useful information about the future pay-off of asset returns and thus should enter the bivariate predictive regression model, and also to avoid data snooping concerns, we resort to the recently proposed method in Rapach, Strauss, and Zhou (2010) who consider a combination of forecasts that assign equal weight to forecasts from individual predictors,

$$\mu_{i,t+1}^{\text{POOL-AVG}} = \frac{1}{K}\sum_{i=1}^{K} \mu_{i,t+1}. \tag{6}$$

The authors show that this simple equally-weighted combination forecasts of stock excess returns outperform the benchmark historical average forecast. The forecast defined in Equation (6), together with forecast for the variance-covariance matrix of returns, are subsequently used to compute the time t vector of optimal portfolio weights defined in Equation (2), and the month $t+1$ return on the portfolio strategy defined in Equation (3) is computed using actual returns data for the current month. This procedure generates a $T - n_1$ time series of one-month ahead out-of-sample portfolio returns.

Since our focus is on the economic value of return predictability for asset allocation with commodities, we use a relatively simple variance-covaraince matrix model of returns: the shrinkage estimator proposed in Ledoit and Wolf (2003). Following Campbell and Thompson (2008), Rapach et al. (2010), among others, we assume that the investor uses a 5-year moving window of past returns to compute the variance-covariance matrix of portfolio returns.

2.3. The Economic Value of Commodities

To assess the economic value of commodities in asset allocation when returns are predictable, we evaluate the performance of a portfolio augmented by commodities against one that includes only traditional asset classes using a number of standard metrics as in DeMiguel, Garlappi, and Uppal (2009), among others.

The first is the Sharpe ratio, λ, for the portfolio,

$$\lambda = \frac{\hat{\mu}_p}{\hat{\sigma}_p}. \tag{7}$$

where $\hat{\mu}_p$ and $\hat{\sigma}_p$ are the mean and variance, respectively, of portfolio excess returns over the forecast evaluation period. We test for the differences in Sharpe ratios of the commodity augmented portfolio and the traditional portfolio using the bootstrap procedure in Ledoit and Wolf (2008).

Second, we compute the certainty equivalent returns for the portfolio, (CER),

$$\text{CER} = \hat{\mu}_p - \frac{1}{2}\gamma\hat{\sigma}_p^2, \tag{8}$$

where $\hat{\mu}_p$ and $\hat{\sigma}_p$ are the mean and variance, respectively, of portfolio returns over the forecast evaluation period, and γ is the coefficient of relative risk aversion. The CER can be interpreted as the risk-free rate of return that an investor with a mean-variance utility function is willing to accept rather than adopting a risky portfolio.

Finally, we consider the average portfolio turnover measure which quantifies the amount of trading required to implement each of the strategies. The average turnover metric, τ, which can be interpreted at the average fraction of portfolio value traded each period, is defined as the average of the sum of absolute change in portfolio weight for each strategy p over the $T - n_1$ rebalancing periods across the assets constituting the investment opportunity set:

$$\tau = \frac{1}{T - n_1} \sum_{t=1}^{T-n_1} \sum_{i=1}^{N} \left(|w_{p,i,t+1} - w_{p,i,t^+}| \right), \tag{9}$$

where $w_{p,i,t}$ and $w_{p,i,t+1}$ are respectively, the portfolio weights in asset i at time t and $t+1$ under strategy k, and $\hat{w}_{p,i,t^+}$ is the portfolio weight before rebalancing at time $t+1$. $|w_{p,i,t+1} - w_{p,i,t^+}|$ reflects the magnitude of trade required by asset i at the rebalancing point $t + 1$. We also study how proportional transactions costs generated by the turnover affects the returns of the portfolio strategies. We set medium and low proportional transaction costs of 50 basis points (bps) and 10bps, respectively. The wealth at the end of time $t + 1$ after paying transactions cost is

$$W_{t+1}^{\text{NTC}} = W_t(1 + R_{p,t+1}) \left(1 - c \sum_{i=1}^{N} |w_{p,i,t} - w_{p,i,t^+}| \right), \tag{10}$$

where c is the unit transactions cost incurred at the end of period $t + 1$. The portfolio returns net of transactions costs is therefore

$$R_{p,t+1}^{\mathrm{NTC}} = \frac{W_{t+1}^{\mathrm{NTC}}}{W_{t}^{\mathrm{NTC}}} - 1. \tag{11}$$

3. Data and Preliminary Analysis

3.1. Asset Classes and Predictor Variables

Our empirical analysis is based on monthly data on widely used market indices of stocks, bonds, and commodity futures spanning the period from January 1976 to December 2015. The asset classes are the value-weighted return on the S&P 500 stock index obtained from the Center for Research in Security Prices (CRSP), the total return on the Barclays Capital U.S. aggregate bond and the S&P GSCI commodity indices both obtained from Bloomberg, and the return on the one-month risk-free Treasury bills obtained from the website of Professor Kenneth French.[5] All values are denominated in United States Dollars. The asset classes are selected so as to make our analysis comparable to the literature that studies the economic value of alternative asset classes for risk-averse investors. We compute simple excess returns on stocks as the value-weighted return less the return on the risk-free bills, and the return on bonds and commodities as the monthly total return relatives less the return on the risk-free bills. The starting date for the sample period is dictated by the availability of data.

Although financial theory does not provide enough guidance on which variables are important, our economic variables include predictors that have been used in the asset return predictability literature for decades to forecast the excess returns on stocks, bonds and commodities. Specifically, we consider a set of 24 macroeconomic and financial variables. Out of these, the first ones obtained from the webpage of Professor Amit Goyal:[6] are the 14 predictors analysed in Welch and Goyal (2008) and extended till December 2015.

1. Log dividend-price ratio (DP): difference between the log of the 12-month moving sum of the dividends paid on the S&P 500 index and the log price of the S&P 500 index.
2. log dividend yield (DY): log of the 12-month moving sum of dividends paid on the S&P 500 index less the log of lagged stock prices on the S&P 500 index.
3. Log earnings-price ratio (EP) is the log of the 12-month moving sum of earnings on the S&P 500 index less the log stock prices of the S&P 500 index.
4. Log dividend-payout ratio (DE): log of 12-month moving sum of dividends less the log of a 12-month moving sum earnings.
5. Excess stock return volatility (RVOL): computed using a 12-month moving standard deviation as in Mele (2007).
6. Book-t-market ratio (BM): book-to-market value ratio for the Dow Jones Industrial Average.
7. Net Equity Expansion (NTIS): ratio of a 12-month moving sum of net equity issues by NYSE-listed stocks to the total end-of-year market capitalization of New York Stock Exchange (NYSE) stocks.

[5]See http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
[6]See http://www.hec.unil.ch/agoyal/.

8. Treasury bill rate (TBL): interest rate on three month Treasury bill (secondary market).
9. Long-term yield (LTY): long term government bond yield
10. Long-term return (LTR): return on long-term government bonds.
11. Term spread (TMS): long term government bond yield minus treasury bill rate.
12. Default yield spread (DFY): difference between yields on Moody's BAA- and AAA-rated corporate bonds.
13. Default return spread (DFR): long-term corporate bond minus long-term government bond returns.
14. Inflation (INFL): log growth in the consumer price index.

The second set of the economic variables include 4 macroeconomic variables assumed to measure the broad state of the economy and are analysed in Gargano and Timmermann (2014) and Hong and Yogo (2012). Specifically, we consider:

15. Industrial production (INDPRO): monthly log growth in industrial production obtained from the website of the Archival Federal Reserve Bank of St. Louis Economic Data (ALFRED).[7]
16. Unemployment rate (UNRATE): monthly unemployment rate from the website of the Archival Federal Reserve Bank of St. Louis Economic Data (ALFRED).
17. Real activity (REA): Kilian (2009) real global economic activity index which drives perceptions abour global ecomic activity.[8]
18. Chicago Fed National Activity index (CFNAI) from ALFRED.

The final set is 5 commodity currencies studied in Gargano and Timmermann (2014) and Chen, Rogoff, and Rossi (2010).

19-24. Log US dollar against five commodity currencies obtained from Bloomberg
   i. Australia (USDAUD)
   ii. Canada (USDCAD)
   iii. New Zealand (USDNZD)
   iv. South Africa (USDZAR)
   v. India (USDIND)

Table 1 reports the summary statistics of monthly returns on the four asset classes and the predictor variables for the full sample period. Panel A shows that stocks recorded the highest return followed by bonds, commodities and then T-bills. The returns on commodity futures are more volatile compared to the other asset classes. Stocks record almost twice the average returns of commodity futures although there is no significance difference in their volatilities. On a risk-adjusted basis, bonds achieved the highest Sharpe ratio of approximately 0.50 per annum, with Commodity futures the lowest with an annualized Sharpe ratio of -0.02. The low mean return and comparatively higher standard deviation make commodity futures unattractive as a stand-alone investment on a risk adjusted basis. The first order sample autocorrelation are all insignificant indicating little linear dependence in returns except for commodity futures, in which case the autocorrelation is significant at the 5% level.

[Insert Table 1 near here]

[7] See https://alfred.stlouisfed.org/.

[8] See http://www-personal.umich.edu/~lkilian/reaupdate.txt.

Panel B reports the cross-market correlation of asset returns. The correlation is positive between commodity futures and risk-free T-bills, negative for bonds, and significantly positive for stocks. The significantly low and negative correlation of commodity futures with the other asset classes suggests that, although commodity futures as a stand-alone investment may not be attractive when compared with the other risky assets, it might provide risk diversification benefits when included in a portfolio composed of stocks, bond and T-bills.

[Insert Table 2 near here]

3.2. Predictive Regression Analysis

3.2.1. Tests of In-sample Return Predictability

To analyse to what extent excess returns on the asset classes are predictable by the economic variables, we follow the conventional framework used by the vast literature on return predictability by estimating the bivariate predictive regression as given in Equation (4)

$$r_{i,t} = \alpha_i + \beta_i x_{j,t-1} + \varepsilon_{i,t}, \tag{12}$$

where $r_{i,t}$ is the log excess return on the risky asset from time $t-1$ to t, $x_{i,t}$ is a predictor variable available at time $t-1$, and ε_t is a zero-mean error term. We estimate the regression by ordinary least squares (OLS). Under the null hypothesis of no predictability, $\beta_i = 0$, and the expected return model reduces to a constant.

Tables 3 reports the estimates of β_i as well as heteroskedasticity-consistent t-statistics and R^2 statistics for log stocks, and bonds and commodities excess returns, respectively. Results are reported for the full sample (February 1976-December 2015). Since we are also interested in gauging the relative strength of predictability during the business cycle, we compute the following version of the conventional R^2 statistic for NBER-dated business cycle expansions (EXP) and recessions (REC):

$$R_c^2 = 1 - \frac{\sum_{t=1}^{T} I_t^c \hat{\varepsilon}_{i,t}^2}{\sum_{t=1}^{T} I_t^c (r_t - \bar{r})^2} \quad \text{for} \ \ c = \text{EXP}, \ \text{REC}, \tag{13}$$

where I_t^{EXP} (I_t^{REC}) is an indicator function that takes a value of unity when t is an expansion (recession) and zero otherwise, $\hat{\varepsilon}_{i,t}$ is the fitted error based on full-sample estimates of the predictive regression model in Equation (12) $\bar{r}$ is the full-sample mean of r_t, and T is the number of observations for the full sample.

From Panel A of Table 3, none of the predictor variables considered for stock excess returns display significant predictive ability at conventional levels for the full sample. The R^2 statistics in the third column of the table are also very small. However, Campbell and Thompson (2008) argue that a monthly R^2 close to 0.5% can still represent an economically significant degree of stock return predictability. This is exceeded by the R^2 of only two predictors namely LTR and DFR. This result re-echoes the findings of Welch and Goyal (2008), and much of the studies on stock return predictability, who find that forecasts based on individual predictors fail to consistently outperform the historical average forecast. The last two columns of Table 3 indicates higher stock excess return predictability during recessions compared to the expansionary periods of the business cycle for all predictors considered except TBL and LTR.

[Insert Table 3 near here]

The results for log bond excess returns are reported in Panel B of Table 3. At conventional levels, YS, CFNAI, and COMMODITY display significant predictive ability, with R^2 statistics of approximately 2% or more. Similar to the results for the log stock excess returns, there is substantially higher predictability during recessions relative to expansions for a number of the predictor variables.

Panel C of Table 3 reports the results for log commodity excess returns. CFNAI show significant predictive ability at the conventional levels. Again similarly to the results for the log stock and bond excess returns, predictability is strongest during recessions relative to expansions for almost all the predictor variables.

To summarize the in-sample evidence of return predictability, we find that most of the economic variables considered for the various asset classes are statistically insignificant confirming the results of prior literature that it is difficult if not impossible to forecast asset returns. However, return predictability is substantially higher during recessions vis-à-vis expansions of the business cycle. In-sample statistical evidence of predictability, while useful, forms only a small part of the story. The true extent of predictability can only be assessed in formal out-of-sample tests and the economic value of such predictability for investor's asset allocation decisions.

3.2.2. Out-of-Sample Return Predictability

The results of the in-sample tests of predictability reported in Table 3 are not true ex-ante measures of future expected returns and would not have been available to the investor in real time because we use the full sample data for estimation. To circumvent this problem, we report measures of out-of-sample predictability for each of the forecast based on the individual economic variables and the combination forecast.

We use data from 1976:02-1989:12 as our initial in-sample estimation period (167 observation), with data from 1990:01-2015:12 serving as the out-of-sample (312 observation) evaluation period. The choice of length of the in-sample estimation period enables us to have a sufficiently long out-of-sample forecasts evaluation period. Hansen and Timmermann (2012), for example, find that using a relatively large proportion of the available sample for forecast evaluation provides better size properties of the test statistics of predictive ability.

We analyse out-of-sample return predictability by comparing the forecasts based on the individual predictors and the combination forecast given in Equations (5) and (6), respectively, to the historical average forecast:

$$\hat{r}_{i,t+1}^{\text{Hist. avg}} = (1/t)\sum_{s=1}^{t} r_{i,t}, \quad i = 1, ..., N. \tag{14}$$

The mean squared forecast error (MSFE) for the ith forecast over the $T - n_1$ forecast evaluation period is given by

$$\text{MSFE}_i = \frac{1}{T - n_1} \sum_{s=1}^{T-n_1} (r_{i,n_1+s} - \mu_{i,n_1+s}). \tag{15}$$

We conduct the analysis using the Campbell and Thompson (2008) out-of-sample R^2 (R^2_{OS}) and the Clark and West (2007) MSFE-adjusted statistics. The R^2_{OS} statistic measures the proportional reduction in MSFE for the predictive regression forecast relative to the historical average forecast,

$$R^2_{\text{OS}} = 1 - (\text{MSFE}_i - \text{MSFE}_0), \tag{16}$$

where MSFE_0 is the MSFE of the the historical average forecast. The historical average is a popular benchmark forecast which assumes a constant expected excess return, that is $\beta_i = 0$ in Equation (5) implying that returns are not predictable, and has been used in many studies on predictability including Goyal and Welch (2003), Welch and Goyal (2008), and Campbell and Thompson (2008). A positive R^2_{OS} indicates that the predictive regression forecast is more accurate than the historical average forecast in terms of MSFE and vice versa. The MSFE-adjusted statistic, on the other hand, tests the null hypothesis that the MSFE of the historical average forecast is less than or equal to the MSFE of the predictive regression forecast against the one-sided alternative hypothesis that the the MSFE of the historical average is greater than the MSFE of the predictive regression forecast.

Panel A of Table 4 presents the out-of-sample results for the log excess stock. Panel A1 of the table report results for the predictive regression forecasts based on the individual economic variables. Although all the R^2_{OS} statistics in the third column are negative, their MSFEs are significantly lower than the MSFE of the historical average forecast indicating that the forecasts based on the individual economic variables outperform the historical average forecast in terms of MSFE. The result for the POOL-AVG forecast, the equally-weighted average of the individual predictors, is reported in Panel A1. This combination forecast has been found to perform well relative to the historical average forecast. Similarly to the results for the individual forecasts, this forecast also outperforms the historical average across the different out-of-sample periods and the business cycle.

Panel B of Table 4 presents results for log bond excess returns. The R^2_{OS} for all the individual predictors reported in Pane B1 is positive, ranging from 1.35% to 1.58% for the full out-of-sample period. That notwithstanding, the MSFEs are significantly lower than the historical average forecast MSFE at conventional levels according to the MSFE-adjusted statistics reported in the fourth column of Table 5. Interestingly, at conventional levels, the MSFE-adjusted statistic indicates that the MSFE for CFNAI and COMMODITY is statistically less than the MSFE of the historical average forecast although the R^2_{OS} is negative. As noted by Clark and West (2007) and McCracken (2007), this is possible when comparing nested model forecasts. The results for the POOL-AVG forecast is reported in Panel B2 of Table 5. The R^2_{OS} statistic for this combination forecast is very high at 3%. The MSFE is significantly lower than the MSFE of the historical average forecast at conventional levels according to the MSFE-adjsuted statistic. Similarly to the in-sample results, predictability is stronger during recessions relative to expansions of the business cycle. Generally, similar results are reported for the subsample periods.

Table 5 presents the results fo log commodity excess returns. Three of the R^2_{OS} statistics for the individual predictors (UNRATE, CFNAI, USDINR) are positive ranging from 0.02% to 2.10%. At the conventional levels, the MSFEs are statistically smaller than MSFE of the historical average forecast in terms of the MSFE-adjusted statistic. This is also the case for almost all the other predictors even though negative R^2_{OS} are documented. The POOl-AVG forecast also statistically outperforms the historical average forecast in terms of MSFE with an R^2_{OS} statistic of 0.06%. Similarly to the in-sample and results reported for the log stock and bond excess returns, predictability is stronger during NBER-dated business cycle expansions and recessions.

4. Out-of-Sample Portfolio Performance

We conduct further out-of-sample testing within an asset allocation framework by measuring the economic value of commodities in the presence of time-varying expected returns to risk-averse investors. We compare the performance of traditional and commodity portfolios using standard performance metrics. The out-of-sample portfolio analysis starts from January 1990 for the full and first out-of-sample periods, and January 2002 for the second out-of-sample period. Similar to the full sample, we use the first 120 observations to initialize the parameter estimates and produce the forecast for the subsequent month for the subsample periods as detailed in Section 2.2.

Tables 6 and 7 report the out-of-sample portfolio performance results for the traditional and commodity portfolios using the Sharpe ratio, certainty equivalent return (CER) gain, and average portfolio turnover as performance metrics. Results are reported for the full out-of-sample periods (1990:01-2015:12) and the two out-of-sample sub period (1990:01-2002:12, 2003:12-2015:12), as well as for the NBER-dated business cycle recessions and expansions. Each line of the table compares the results of the commodity portfolio and the traditional portfolio. The comparison differs by constraints on portfolio weights: restricting short sales and leverage to 50% of wealth ($-0.5 < \boldsymbol{w}_t < 1.5$), precluding short sales and limiting leverage to 50% of wealth ($0 \leq \boldsymbol{w}_t \leq 1.5$), precluding short sales and leverage ($0 \leq \boldsymbol{w}_t \leq 1$); and the level coefficient of relative risk aversion ($\gamma = 2, 5, 10$). Since the investor will rebalance her investor every month thus incurring cost from trading, we set a proportional transactions cost of 50 bps for each risky asset class. These choices are made following the vast literature on asset allocation.

4.1. Sharpe ratio

Panel A of Table 6 reports the Sharpe ratios for the full out-of-sample period. Across all portfolio weight constraints and investor's risk preferences, the commodity portfolio generate higher Sharpe ratios than the traditional portfolio. Imposing transactions cost of 50 bps results in negative Sharpe ratios. The commodity portfolio continues to dominate the traditional portfolio, however. For example, for $\boldsymbol{w}_t \in [-0.5, 1.5]$ and moderate risk aversion, $\gamma = 5$, the annualized Sharpe ratio (net-of-transactions-costs Sharpe ratio) for the commodity portfolio is 0.66 (-0.08) compared to 0.59 (-0.24) for the traditional portfolio. The Sharpe ratios for both commodity and traditional portfolios are much higher during expansions than in recessions. Considering that the evidence of predictability was much stronger during recessions as detailed in Section 3.2, one would have expected higher Sharpe ratios during this part of the business cycle. Again, the Sharpe ratios during expansionary business cycle are substantially higher vis-à-vis the overall period. The opposite is also true when compared to recessions.

The results for the two out-of-sample sub periods are reported in Panels B and C of Table 6. Similarly to the conclusions for the full out-of-sample period, the commodity portfolio continues to dominates the traditional portfolio across portfolio weight constraints, risk preferences, and the business cycle. In fact, most of the net-of-transactions-costs Sharpe ratio are positive unlike the negative values realized for the full sample. There are, however, a few striking results worth highlighting. For the first subsample period, investors who considered a commodity portfolio when the economy was in expansionary period would have underperformed those investors who considered a traditional portfolio. Conversely, a commodity portfolio for the same period with the economic in recession would have outperformed a traditional portfolio. For example, during a recessions, and for $\gamma = 5$ and $\boldsymbol{w}_t \in [-0.5, 1.5]$, the commodity portfolio records an annualized net-of-transactions-costs Sharpe ratio of 0.20 compared to a -1.44 for the traditional portfolio. We report similar results when considering the second subsample where we see that a traditional portfolio dominates a commodity portfolio during expansions and vice versa.

[Insert Table 6 near here]

Considering that our portfolio strategy is dynamic, the Sharpe ratio has its limitation as a performance measure in the presence of time-varying expected returns and time-varying volatility. This is because unconditional volatility is not an appropriate measure of the risk faced by the investor. We therefore consider an alternative measure of performance, the CER gain, which is robust to the concerns raised.

4.2. Certainty Equivalent Return Gain

Table 7 reports the annualized percent CER gain, and the net-of-transactions-costs CER gain assuming a proprtional transactions cost of 50 bps for varying levels of risk aversion and portfolio weight constraints. The CER gain is the difference between the CER for the commodity and traditional portfolios defined in Equation (8). If this difference is positive, then the commodity portfolio is said to dominate the traditional portfolio.

[Insert Table 7 near here]

Panel A of the presents the results for the full out-of-sample period. Across almost portfolio weight constraints and risk preferences, the CER gains are positive. The gains assuming a relative risk aversion of 5, and limiting short sales and leverage constraint to 50% of wealth, for example, amounts to 90 basis points.

As shown in the table, the commodity portfolio turn over approximately one time more than the traditional portfolio. Despite this low turnover, the net-of-transactions-costs CER gains are negative under most of the portfolio weights constraints and levels of risk aversion coefficient.

The exception is when we limit short sales and leverage to 50% of wealth ($\boldsymbol{w}_t \in [-0.5, 1.5]$) and for a risk preferences of 5, which leads to net-of-transactions-costs CER gain of 133 basis points. For the same risk aversion and portfolio weight constraint, we realise a CER gain net of cost of 1338 basis points during recessions and a CER loss of 51 basis during expansions. These results suggest that imposing sensible restrictions on portfolio weights could lead to better portfolio performance as evidenced in Jagannathan and Ma (2003). The substantially higher net-of-cost CER gains in recessions relative to expansions is inline with our out-of-sample R^2_{OS} test of predictive ability.

The results for the two out-of-sample sub periods deliver similar results to the full sample. Under moderate risk aversion of 5 and portfolio weight restrictions $\boldsymbol{w}_t \in [-0.5, 1.5]$, the net-of-transactions-costs CER gain is 194 basis points for the first subsample and 55 basis for the second subsample. In fact, these are the only cases where the net-of-transactions-costs CER are positive for the first subsample period re-echoing the role of investor's risk preferences and sensible restrictions on portfolio weights. Similarly to the full sample period, results during recessions are stronger relative to expansions for the first sample. The second sub sample which happens to include the 2000s commodity boom period, has reports CER gains before and after deducting transactions cost during recessions, and CER loss during expansions. This holds across all risk preferences and portfolio constraints. For example, for a relative risk aversion coefficient of 5 and portfolio weights constraints of $\boldsymbol{w}_t \in [-0.5, 1.5]$, the CER (net-of-transactions-costs) gain is 2216 (1795) basis points during recessions compared to CER (net-of-transactions-costs) loss of 169 (227) basis points during expansions.

The out-of-sample portfolio performance analysis demonstrates the economic value of commodities in asset allocation in the presence of time-varying expected returns, with results especially concentrated during the recessionary phase of the business cycle.

5. Conclusion

We utilize recently developed methods for improving forecasts of asset returns to re-examine the economic value of commodities in asset allocation in the presence of time-varying expected returns. We utilize a recursive estimation approach, where the parameters of the return prediction models are updated by adding one observation as new data becomes available. Data from 1976:02-1989:12 are used to initialize the parameter estimates. This procedure generates pseudo out-of-sample forecasts of excess returns which are subsequently used in the asset allocation exercise. The out-of-sample forecast evaluation period is 1990:01-2015:12.

Our in-sample and out-of-sample tests of predictability shows that whiles majority of the individual economic variables do not display statistically significant predictability for for monthly log excess returns at conventional levels, recently developed methods of forecasts combination in Bates and Granger (1969) and Rapach et al. (2010) deliver statistically significant predictive ability out-of-sample.

In portfolio performance analysis using the Sharpe ratio, certainty equivalent return gain, and portfolio turnover indicate substantial economic value of commodities in asset allocation to a mean-variance investor who, based on the evidence of return predictability from macroeconomic and financial variables, combine forecasts from individual bivariate predictive regressions. Our results also show a the role for the business-cycle. Specifically, we find net-of-transactions-costs certainty equivalent gains during recessions relative to expansions for the full out-of-sample and second out-of-sample periods, and expansions relative to recessions for the first out-of-sample period. These results, however, are subject to the use of moderate investor risk aversion and imposing sensible constraints on portfolio weights. Overall, our results reverse the conclusions in Daskalaki and Skiadopoulos (2011) and to some extent Bessler and Wolff (2015) who assume a constant expected returns model in their asset allocation exercise.

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Table 1
Summary Statistics for Monthly Log Returns and Predictor Variables

This table reports the summary statistics for monthly log returns on stock, bond and commodity indices, T-bills (in percent), and financial and macroeconomic variables. TBL, LTR, TMS, DFY, DFR, INFL, INDPRO, USDAUD, USDCAD, USDNZD, USDZAR, USDINR are measured in percent. Panel A and B reports the percent mean, percent standard deviation, minimum and maximum values, the first order autocorrelation, and the annualized Sharpe ratio. The cross-market correlation of returns is reported in Panel C. *** indicates significance at the 1% level. The sample period is February 1976-December 2015 (479 observations)

Variable	Mean	Std. dev.	Min	Max	Auto-correlation	Sharpe ratio
Panel A: Asset Classes						
T-bills	0.39	0.29	0.00	1.34	0.97	0.00
Stocks	0.87	4.47	−25.54	12.09	0.08	0.37
Bonds	0.61	1.55	−6.27	10.74	0.17	0.50
Commodities	0.36	5.64	−33.13	20.65	0.18	−0.02
Panel B: Predictor Variables						
DP	−3.65	0.44	−4.52	−2.75	0.99	
DY	−3.64	0.44	−4.53	−2.75	0.99	
EP	−2.85	0.49	−4.84	−1.90	0.99	
DE	−0.80	0.35	−1.24	1.38	0.98	
RVOL	0.15	0.05	0.05	0.32	0.96	
BM	0.46	0.28	0.12	1.21	0.99	
NTIS	0.01	0.02	−0.06	0.05	0.98	
TBL	4.79	3.56	0.01	16.30	0.99	
LTY	0.07	0.03	0.02	0.15	0.99	
LTR	0.75	3.19	−11.24	15.23	0.04	
TMS	2.21	1.47	−3.65	4.55	0.95	
DFY	1.10	0.46	0.55	3.38	0.96	
YS	2.98	1.54	−2.28	5.93	0.97	
DFR	−0.01	1.46	−9.75	7.37	−0.03	
INFL	0.30	0.32	−1.77	1.43	0.65	
REA	−0.44	24.44	−61.77	66.08	0.95	
CFANI	−0.03	0.93	−4.63	2.72	0.62	
INDPRO	0.18	0.68	−4.40	2.07	0.28	
UNRATE	6.44	1.56	3.80	10.80	0.99	
USDAUD	−0.11	3.30	−18.68	9.92	0.03	
USDCAD	−0.07	1.99	−13.03	8.85	−0.06	
USDNZD	−0.09	3.50	−24.89	18.01	−0.03	
USDZAR	−0.60	4.17	−24.82	14.05	0.03	
USDINR	−0.42	2.12	−19.89	7.05	0.06	

Panel C: Cross-market Correlations

	T-bills	Stocks	Bonds	Commodity
T-bills	1			
Stocks	0.011	1		
Bonds	0.151***	0.200***	1	
Commodities	0.072	0.228***	−0.018	1

Table 2

Summary Statistics and Correlation for Monthly Log Returns during the Business Cycle

This table reports the summary statistics for monthly log returns on stock, bond and commodity indices, T-bills (in percent) for the NBER-dated bsuiness cycle expansions and recessions. Panel A and B reports the percent mean, percent standard deviation, minimum and maximum values, the first order autocorrelation, and correlation matrix. ** and *** indicate significance at the 5% and 1% levels. The sample period is February 1976-December 2015 (479 observations)

Variable	Mean	Std. dev.	Min	Max	Auto-correlation	Sharpe ratio	T-bills	Stocks	Bonds	Commodities
					Panel A: Economic Expansion					
T-bills	0.37	0.27	0.00	1.34	0.97	0.00	1			
Stocks	1.04	4.07	−25.54	12.09	−0.04	0.57	0.017	1		
Bonds	0.56	1.30	−6.27	5.09	0.08	0.50	0.113***	0.157**	1	
Commodities	0.52	5.04	−15.56	15.61	0.06	0.10	0.057	0.156**	−0.039	1
					Panel B: Economic Recession					
T-bills	0.53	0.40	0.00	1.27	0.97	0.00	1			
Stocks	−0.26	6.51	−20.41	11.24	−0.04	−0.42	0.062	1		
Bonds	0.99	2.68	−6.10	10.74	0.08	0.60	0.197	0.331**	1	
Commodities	−0.72	8.72	−33.13	20.65	0.06	−0.50	0.166	0.389**	0.042	1

Table 3
In-sample Predictive Regression Estimation Results

This table reports the in-sample estimation results for the bivariate predictive regression model of log stock,bond and commodity excess returns and the economic variables individually. The square bracket to the immediate right of slope coefficients report the heteroskedasticity-consistent t-statistic. The R^2 statistics are computed for the full sample (February 1976-December 2015). The R^2_{EXP} (R^2_{REC}) statistics in the fourth (fifth) columns are computed for NBER-dated business-cycle expansions (EXP) and recessions (REC), as given by Equation (13) in the text.

Economic variable	Slope Coefficient	R^2 (%)	R^2_{EXP} (%)	R^2_{REC} (%)
Panel A: Results for log excess stock returns				
DP	0.50 [1.03]	0.24	−0.13	0.87
DY	0.56 [1.16]	0.31	−0.20	1.15
EP	0.28 [0.54]	0.09	0.08	0.11
DE	0.25 [0.33]	0.04	−0.08	0.23
RVOL	5.74 [1.55]	0.40	−0.34	1.64
BM	0.31 [0.40]	0.04	−0.09	0.25
NTIS	1.26 [0.10]	0.00	−0.05	0.09
TBL	-0.05 [-0.86]	0.17	0.29	−0.02
LTY	-0.04 [-0.59]	0.08	0.22	−0.16
LTR	0.10 [1.42]	0.54	0.36	0.83
TMS	0.14 [0.96]	0.20	0.00	0.53
DFY	0.09 [0.16]	0.01	0.01	0.01
DFR	0.32 [1.38]	1.11	0.63	1.93
INFL	0.06 [0.08]	0.00	−0.09	0.16
Panel B: Results for log excess bond returns				
TBL	-0.02 [-0.47]	0.12	0.62	−0.70
TMS	0.15 [1.96]	2.06	3.51	−0.35
DFY	0.20 [1.01]	0.36	−0.50	1.81
YS	0.16 [2.25]	2.72	4.01	0.58
CFNAI	-0.25 [-2.72]	2.41	1.59	3.79
COMMODITY	-0.04 [-2.56]	1.85	0.65	3.87
Panel C: Results for log excess commodity returns				
RVOL	2.31 [0.44]	0.04	0.00	0.12
TBL	0.04 [0.47]	0.05	0.29	−0.37
DFY	-0.77 [-1.13]	0.40	−0.10	1.25
YS	-0.12 [-0.70]	0.12	0.45	−0.47
INFL	0.46 [0.47]	0.07	0.02	0.16
INDPRO	0.41 [0.78]	0.25	−0.42	1.39
UNRATE	-0.92 [-1.54]	0.56	0.16	1.23
REA	0.01 [0.95]	0.27	0.75	−0.54
CFNAI	0.92 [2.35]	2.35	1.01	4.67
USDAUD	0.08 [0.73]	0.20	−0.88	2.06
USDCAD	0.06 [0.33]	0.04	−0.36	0.73
USDNZD	0.07 [0.80]	0.18	−0.52	1.39
USDZAR	0.05 [0.68]	0.14	−0.91	1.94
USDINR	0.19 [1.31]	0.49	0.10	1.16

Table 4
Out-of-Sample Forecasting Results for Excess Stock and Bond Returns

This table reports the estimation results for the bivariate predictive regression model of log excess stock and bond returns and one of the respective financial and macroeconomic variables. The square bracket to the immediate right of slope reports the heteroskedasticity-consistent t-statistic. The R^2_{OS} statistics are computed for the full sample out-of-sample period (1990:01-2015:12) and two out-of-sample sub periods (1990:01-2002:12 and 2003:01-2015:12). The R^2_{OS} statistics in the fourth, eighth and twelfth (fifth, ninth and thirteenth) columns are computed for NBER-dated business-cycle expansions (EXP) and recessions (REC), as given by (9) in the text. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively, based on one-sided (upper-tail) wild bootstrapped p-values.

	1990:01-2015:12 out-of-sample period					1990:01-2002:12 out-of-sample period					2003:01-2015:12 out-of-sample period				
Economic variable	MSFE	R^2_{OS} (%)	MSFE-adjusted	R^2_{OS} EXP (%)	R^2_{OS} REC (%)	MSFE	R^2_{OS} (%)	MSFE-adjusted	R^2_{OS} EXP (%)	R^2_{OS} REC (%)	MSFE	R^2_{OS} (%)	MSFE-adjusted	R^2_{OS} EXP (%)	R^2_{OS} REC (%)
Panel A: Stock Index Excess Returns															
HIST. AVG	19.27					20.41					18.12				
Panel A1: Bivariate predictive regression forecasts															
DP	19.45	−0.97	−0.49*	−1.93	1.27	20.64	−1.12	−0.16**	−1.84	1.93	18.27	−0.79	−0.87	−2.07	0.93
DY	19.46	−1.03	−0.39*	−2.38	2.11	20.69	−1.40	−0.22**	−2.49	3.24	18.23	−0.60	−0.47	−2.20	1.55
EP	19.44	−0.91	−0.34**	−1.03	−0.62	20.42	−0.05	0.30***	−0.16	0.40	18.46	−1.86	−0.53	−2.41	−1.13
DE	19.60	−1.75	−0.69***	−0.93	−3.65	20.63	−1.07	−1.52*	−1.25	−0.31	18.58	−2.51	−0.42*	−0.43	−5.33
RVOL	19.29	−0.10	0.65***	−0.32	0.41	20.58	−0.82	−0.21**	−1.22	0.88	18.00	0.71	1.29**	1.11	0.17
BM	19.34	−0.41	−0.96***	−0.72	0.31	20.52	−0.56	−0.71***	−0.64	−0.22	18.17	−0.24	−0.78*	−0.85	0.58
NTIS	19.70	−2.24	−1.65***	−1.90	−3.04	20.87	−2.25	−0.79***	−2.73	−0.18	18.53	−2.23	−1.71***	−0.56	−4.48
TBL	19.41	−0.77	−0.29*	−0.32	−1.82	20.66	−1.25	−0.41	−1.30	−1.07	18.17	−0.23	0.12***	1.23	−2.19
LTY	19.41	−0.72	−0.54*	−0.49	−1.26	20.63	−1.10	−0.46	−0.96	−1.67	18.18	−0.30	−0.33***	0.25	−1.05
LTR	19.31	−0.21	0.39***	−0.65	0.83	20.52	−0.54	−0.03**	−0.83	0.73	18.09	0.16	0.49**	−0.37	0.88
TMS	19.33	−0.32	0.07***	−0.38	−0.17	20.52	−0.55	−0.10*	−1.07	1.71	18.13	−0.05	0.28***	0.74	−1.12
DFY	19.47	−1.05	−1.17**	−1.12	−0.89	20.58	−0.86	−0.37**	−1.78	3.07	18.35	−1.26	−1.33**	−0.06	−2.87
DFR	19.60	−1.75	0.18***	−1.28	−2.83	21.00	−2.88	−0.83**	−2.99	−2.39	18.21	−0.48	0.47**	1.44	−3.06
INFL	19.36	−0.50	−0.91***	−0.11	−1.41	20.45	−0.23	−0.23*	−0.09	−0.85	18.27	−0.81	−0.99**	−0.16	−1.70
Panel A2: Pool-average forecasts															
POOL-AVG	19.33	−0.35	−0.84***	−0.39	−0.26	20.51	−0.48	−1.36**	−0.74	0.61	18.16	−0.21	−0.24**	0.15	−0.69
Panel B: Bond Index Excess Returns															
HIST. AVG	1.08					1.17					0.99				
Panel B1: Bivariate predictive regression forecasts															
TBL	1.09	−0.17	0.27***	−0.18	−0.15	1.18	−0.22	−0.09***	−0.23	−0.13	1.00	−0.11	0.40***	−0.10	−0.16
TMS	1.07	1.58	2.42***	1.26	3.14	1.18	−0.23	1.25***	−0.75	4.26	0.96	3.72	2.24***	4.08	2.59
DFY	1.09	−0.73	0.12**	−1.29	2.03	1.20	−2.19	−0.83**	−2.41	−0.25	0.98	1.00	0.93***	0.30	3.15
YS	1.07	1.35	2.58***	0.71	4.51	1.17	−0.09	1.47***	−0.62	4.53	0.96	3.05	2.16**	2.57	4.50
CFNAI	1.10	−1.58	1.39***	1.98	−19.06	1.15	2.04	1.94***	1.79	4.17	1.05	−5.84	0.41***	2.25	−30.53
COMMODITY	1.19	−9.60	1.22***	−10.15	−6.87	1.34	−14.21	0.84***	−11.85	−34.74	1.04	−4.16	0.88***	−7.77	6.88
Panel B2: Pool-average forecasts															
POOL-AVG	1.05	2.79	2.23	2.34***	4.98	1.15	2.38	1.88***	1.43	10.65	0.96	3.27	1.43***	3.62	2.18

Table 5
Out-of-Sample Forecasting Results for Excess Commodity Returns

This table reports the estimation results for the bivariate predictive regression model of log excess commodity returns and one of the respective financial and macroeconomic variables. The square bracket to the immediate right of slope reports the heteroskedasticity-consistent t-statistic. The R^2 statistics are computed for the full sample out-of-sample period (1990:01-2015:12) and two out-of-sample sub periods (1990:01-2002:12 and 2003:01-2015:12). The R^2_{EXP} (R^2_{REC}) statistics in the fourth, eighth and twelfth (fifth, ninth and thirteenth) columns are computed for NBER-dated business-cycle expansions (EXP) and recessions (REC), as given by (9) in the text. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively, based on one-sided (upper-tail) wild bootstrapped p-values.

	1990:01-2015:12 out-of-sample period					1990:01-2002:12 out-of-sample period					2003:01-2015:12 out-of-sample period				
Economic variable	MSFE	R^2_{OS} (%)	MSFE-adjusted	R^2_{OS} EXP (%)	R^2_{OS} REC (%)	MSFE	R^2_{OS} (%)	MSFE-adjusted	R^2_{OS} EXP (%)	R^2_{OS} REC (%)	MSFE	R^2_{OS} (%)	MSFE-adjusted	R^2_{OS} EXP (%)	R^2_{OS} REC (%)
HIST. AVG	35.73					20.05					47.77				
Panel A: Bivariate predictive regression forecasts															
RVOL	35.93	−0.58	−1.32***	−0.82	−0.05	20.19	−0.68	−0.88***	−0.88	−0.18	48.03	−0.55	−1.05*	−0.81	−0.01
TBL	36.06	−0.92	−0.66	−0.34	−2.18	20.49	−2.16	−0.50	−2.96	−0.16	48.02	−0.52	−0.47	0.55	−2.76
DFY	35.79	−0.18	0.40*	−1.03	1.67	20.24	−0.91	−0.02	−2.57	3.24	47.74	0.06	0.42*	−0.50	1.23
YS	35.80	−0.20	−0.81**	−0.09	−0.43	20.13	−0.37	−1.30***	−0.33	−0.47	47.84	−0.14	−0.44**	−0.01	−0.42
INFL	35.88	−0.42	−0.72**	−0.15	−1.01	20.17	−0.59	−0.64**	−1.00	0.45	47.95	−0.37	−0.54*	0.14	−1.42
INDPRO	35.99	−0.74	−0.68**	−0.01	−2.31	20.06	−0.04	0.49***	1.02	−2.69	48.23	−0.96	−1.22*	−0.36	−2.20
UNRATE	35.72	0.02	0.89**	−2.48	5.46	20.02	0.15	1.04**	−1.47	4.20	47.78	−0.02	0.52**	−2.83	5.82
REA	35.87	−0.39	−0.47**	−0.27	−0.64	20.20	−0.74	−0.50***	−1.25	0.55	47.91	−0.28	−0.28	0.06	−0.97
CFNAI	34.98	2.10	1.60***	−0.21	7.10	19.99	0.32	0.68***	0.58	−0.34	46.50	2.67	1.55***	−0.48	9.21
USDAUD	35.92	−0.53	−0.56**	−0.52	−0.55	20.23	−0.88	−0.73***	−0.76	−1.19	47.97	−0.41	−0.29*	−0.43	−0.36
USDCAD	36.13	−1.13	−1.40**	−1.09	−1.22	20.40	−1.73	−1.50***	−2.10	−0.81	48.22	−0.94	−0.89*	−0.74	−1.34
USDNZD	35.90	−0.48	−0.32**	−1.63	2.02	19.94	0.58	0.77***	−0.79	4.02	48.17	−0.82	−1.03*	−1.92	1.46
USDZAR	35.87	−0.40	−0.32**	−0.09	−1.09	20.04	0.06	0.50***	0.64	−1.39	48.04	−0.55	−0.73*	−0.34	−1.01
USDINR	35.70	0.09	0.61**	−0.18	0.67	20.15	−0.48	−1.86***	−0.58	−0.23	47.64	0.27	0.85*	−0.05	0.93
Panel B: Pool-average forecasts															
POOL-AVG	35.71	0.06	0.37**	−0.34	0.92	20.06	−0.03	0.00***	−0.24	0.48	47.73	0.09	0.38*	−0.37	1.05

Table 6
Measure of Portfolio Performance: Sharpe ratio

This table reports the Sharpe ratio portfolio performance measure for an investor with mean-variance preferences and relative risk aversion coefficients of $\gamma = 2, 5, 10$, who faces the following portfolio weight constraints: restricting short sales and leverage to 50% of wealth ($-0.5 < \boldsymbol{w}_t < 1.5$), precluding short sales and limiting leverage to 50% of wealth ($0 \leq \boldsymbol{w}_t \leq 1.5$), precluding short sales and leverage ($0 \leq \boldsymbol{w}_t \leq 1$). The λ statistic is the annualized Sharpe ratio for the portfolio strategies. The λ statistic is also reported for NBER-dated business cycle expansions and recessions. The λ^{NTC} statistic is the annualized Sharpe ratio assuming a proportional transactions cost of 50 basis points per transaction. τ is the average turnover generated by the portfolios. Results are reported for the full sample out-of-sample period (1990:01-2015:12) and two out-of-sample sub periods (1990:01-2002:12 and 2003:01-2015:12).

	Commodity Portfolio						Traditional Portfolio					
	Overall		Expansion		Recession		Overall		Expansion		Recession	
Portfolio Weights Constraints	Sharpe ratio	Sharpe ratio TC = 50 bps	Sharpe ratio	Sharpe ratio TC = 50 bps	Sharpe ratio	Sharpe ratio TC = 50 bps	Sharpe ratio	Sharpe ratio TC = 50 bps	Sharpe ratio	Sharpe ratio TC = 50 bps	Sharpe ratio	Sharpe ratio TC = 50 bps
Panel A: 1990:01-2015:12 out-of-sample period												
$\gamma = 2$												
$[-0.5, 1.5]$	0.56	−0.09	0.80	0.05	−0.29	−0.65	0.53	−0.11	0.85	0.15	−0.97	−1.42
$[0, 1.5]$	0.53	−0.12	0.77	0.03	−0.35	−0.71	0.54	−0.10	0.85	0.16	−0.96	−1.39
$[0, 1]$	0.50	−0.07	0.75	0.09	−0.37	−0.70	0.53	−0.03	0.86	0.25	−1.02	−1.42
$\gamma = 5$												
$[-0.5, 1.5]$	0.66	−0.08	0.93	0.10	−0.40	−0.85	0.59	−0.24	0.90	0.08	−0.79	−1.55
$[0, 1.5]$	0.63	−0.10	0.89	0.07	−0.40	−0.84	0.61	−0.11	0.91	0.13	−0.72	−1.23
$[0, 1]$	0.58	−0.02	0.83	0.15	−0.40	−0.76	0.56	−0.02	0.87	0.23	−0.83	−1.24
$\gamma = 10$												
$[-0.5, 1.5]$	0.74	−0.32	1.02	−0.18	−0.24	−1.02	0.69	−0.33	1.01	−0.09	−0.48	−1.22
$[0, 1.5]$	0.71	−0.16	0.98	0.00	−0.23	−0.81	0.70	−0.19	1.01	0.02	−0.43	−1.05
$[0, 1]$	0.63	−0.06	0.90	0.13	−0.38	−0.83	0.61	−0.08	0.93	0.17	−0.63	−1.12
Panel B: 1990:01-2002:12 out-of-sample period												
$\gamma = 2$												
$[-0.5, 1.5]$	0.44	−0.20	0.41	−0.26	0.64	0.17	0.45	−0.09	0.55	0.03	−0.64	−1.53
$[0, 1.5]$	0.46	−0.18	0.44	−0.22	0.59	0.13	0.46	−0.07	0.55	0.04	−0.64	−1.46
$[0, 1]$	0.37	−0.23	0.37	−0.25	0.40	−0.08	0.39	−0.10	0.52	0.06	−0.83	−1.45
$\gamma = 5$												
$[-0.5, 1.5]$	0.70	−0.06	0.66	−0.08	1.04	0.20	0.69	−0.21	0.73	0.02	0.25	−1.45
$[0, 1.5]$	0.70	−0.02	0.67	−0.04	1.00	0.23	0.70	0.04	0.74	0.11	0.24	−1.14
$[0, 1]$	0.58	−0.03	0.56	−0.06	0.79	0.22	0.57	0.03	0.63	0.12	−0.20	−1.27
$\gamma = 10$												
$[-0.5, 1.5]$	0.83	−0.45	0.76	−0.43	1.51	−0.64	0.83	−0.37	0.83	−0.24	0.90	−0.96
$[0, 1.5]$	0.83	−0.10	0.76	−0.13	1.51	0.22	0.82	−0.10	0.82	−0.05	0.93	−0.74
$[0, 1]$	0.74	−0.03	0.70	−0.05	1.20	0.21	0.75	−0.02	0.77	0.05	0.55	−0.95
Panel C: 2003:01-2015:12 out-of-sample period												
$\gamma = 2$												
$[-0.5, 1.5]$	0.52	0.00	0.81	0.23	−0.50	−0.86	0.61	−0.01	1.38	0.55	−1.22	−1.52
$[0, 1.5]$	0.50	0.01	0.83	0.26	−0.61	−0.90	0.63	0.00	1.38	0.55	−1.20	−1.51
$[0, 1]$	0.56	0.12	0.88	0.38	−0.61	−0.85	0.70	0.20	1.43	0.78	−1.17	−1.42
$\gamma = 5$												
$[-0.5, 1.5]$	0.62	0.01	0.90	0.20	−0.15	−0.60	0.48	−0.15	1.18	0.34	−1.22	−1.58
$[0, 1.5]$	0.58	0.01	0.92	0.24	−0.43	−0.79	0.51	−0.13	1.18	0.34	−1.11	−1.47
$[0, 1]$	0.49	0.07	0.82	0.33	−0.52	−0.78	0.56	0.09	1.28	0.66	−1.16	−1.40
$\gamma = 10$												
$[-0.5, 1.5]$	0.82	0.14	1.07	0.26	0.29	−0.18	0.56	−0.14	1.25	0.31	−0.94	−1.35
$[0, 1.5]$	0.74	0.09	1.09	0.31	−0.23	−0.65	0.59	−0.12	1.25	0.31	−0.86	−1.26
$[0, 1]$	0.65	0.20	0.99	0.45	−0.35	−0.63	0.48	0.04	1.18	0.59	−1.04	−1.31

Table 7
Certainty Equivalent Return Gain for Commodity vs. Traditional Portfolios

This table reports the certainty equivalent return portfolio performance measure for an investor with mean-variance preferences and relative risk aversion coefficients of $\gamma = 2, 5, 10$, who faces the following portfolio weight constraints: restricting short sales and leverage to 50% of wealth ($-0.5 < \boldsymbol{w}_t < 1.5$), precluding short sales and limiting leverage to 50% of wealth ($0 \le \boldsymbol{w}_t \le 1.5$), precluding short sales and leverage ($0 \le \boldsymbol{w}_t \le 1$). The Δ statistic is the annualized percent certainty equivalent return (CER) gain, the difference between the CER for the commodity and traditional portfolios. The Δ statistic is also reported for NBER-dated business cycle expansions and recessions. Relative turnover is the average turnover for the commodity portfolio divided by the average turnover for the traditional portfolio. The $\Delta^{\text{COST} = 50 \text{ BPS}}$ statistic is the CER gain assuming a proportional transactions cost of 50 basis points per transaction. Results are reported for the full sample out-of-sample period (1990:01-2015:12) and two out-of-sample sub periods (1990:01-2002:12 and 2003:01-2015:12).

	Overall			Expansion			Recession		
Portfolio Weights Constraints	Δ (ann.) (%)	Δ (ann.) cost = 50 bps (%)	Relative turnover	Δ (ann.) (%)	Δ (ann.) cost = 50 bps (%)	Relative turnover	Δ (ann.) (%)	Δ (ann.) cost = TC = 50 bps (%)	Relative turnover
Panel A: 1990:01-2015:12 out-of-sample period									
$\gamma = 2$									
$[-0.5, 1.5]$	1.41	−2.69	1.45	1.13	−3.20	1.47	2.73	0.32	1.30
$[0, 1.5]$	0.47	−3.49	1.45	0.34	−3.76	1.45	0.78	−2.12	1.40
$[0, 1]$	0.67	−2.54	1.70	0.59	−2.70	1.70	0.89	−1.71	1.66
$\gamma = 5$									
$[-0.5, 1.5]$	0.90	1.33	1.03	0.92	−0.51	1.22	0.33	13.38	0.46
$[0, 1.5]$	0.16	−1.44	1.25	0.20	−1.53	1.26	−0.47	−1.27	1.13
$[0, 1]$	0.20	−1.34	1.45	0.19	−1.46	1.47	−0.08	−0.81	1.25
$\gamma = 10$									
$[-0.5, 1.5]$	0.26	−2.96	1.31	0.30	−3.74	1.40	−0.26	2.23	0.98
$[0, 1.5]$	−0.05	−0.75	1.15	−0.02	−0.78	1.17	−0.52	−0.82	1.06
$[0, 1]$	−0.06	−0.72	1.28	−0.01	−0.74	1.31	−0.60	−0.82	1.12
Panel B: 1990:01-2002:12 out-of-sample period									
$\gamma = 2$									
$[-0.5, 1.5]$	−0.34	−6.23	1.72	−3.03	−9.43	1.78	19.67	17.77	1.25
$[0, 1.5]$	−0.09	−6.04	1.75	−2.57	−8.81	1.76	18.33	14.63	1.60
$[0, 1]$	0.06	−4.41	2.07	−2.02	−6.69	2.11	15.57	12.74	1.78
$\gamma = 5$									
$[-0.5, 1.5]$	0.04	1.94	0.94	−1.08	−2.87	1.29	8.36	37.68	0.23
$[0, 1.5]$	0.00	−2.30	1.40	−1.07	−3.58	1.43	7.94	7.24	1.12
$[0, 1]$	0.03	−2.23	1.73	−0.99	−3.48	1.78	7.63	7.11	1.23
$\gamma = 10$									
$[-0.5, 1.5]$	0.00	−5.83	1.45	−0.59	−7.97	1.64	4.55	10.15	0.93
$[0, 1.5]$	0.00	−0.82	1.21	−0.56	−1.49	1.23	4.34	4.22	1.02
$[0, 1]$	−0.08	−0.83	1.32	−0.63	−1.48	1.35	4.05	4.13	1.05
Panel C: 2003:01-2015:12 out-of-sample period									
$\gamma = 2$									
$[-0.5, 1.5]$	−1.10	−5.61	1.47	−1.73	−5.74	1.42	1.73	−6.03	1.85
$[0, 1.5]$	−1.99	−5.71	1.40	−1.17	−5.00	1.40	−8.96	−11.91	1.38
$[0, 1]$	0.66	−2.95	1.74	1.24	−2.55	1.76	−3.90	−6.19	1.53
$\gamma = 5$									
$[-0.5, 1.5]$	1.59	0.55	1.14	−1.69	−2.27	1.08	22.16	17.95	1.46
$[0, 1.5]$	0.66	0.27	1.06	−1.40	−1.88	1.07	13.53	13.75	1.01
$[0, 1]$	−1.25	−2.52	1.35	−1.87	−3.16	1.35	1.95	0.85	1.37
$\gamma = 10$									
$[-0.5, 1.5]$	2.20	2.16	0.99	−0.78	−0.61	0.95	21.48	19.75	1.18
$[0, 1.5]$	1.38	1.83	0.91	−0.64	−0.25	0.91	14.63	15.38	0.89
$[0, 1]$	1.27	1.04	1.11	−0.52	−0.88	1.15	12.82	13.41	0.88