Investing in Global and Emerging Markets: An Application of Integer Programming

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January 2014

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In this analysis of the risk and return of stocks in global markets, we apply several portfolio construction and optimization techniques to U.S. and global stock universes. We find that (1) mean-variance techniques continue to produce portfolios capable of generating excess returns above transactions costs and statistically significant asset selection, (2) optimization techniques minimizing tracking error at risk are statistically significant in portfolio construction; and (3) emerging markets offer the potential for high returns relative to risk. In this experiment, mean-variance tracking error at risk and enhanced index-tracking techniques are examined. Integer programming is necessary if an investor prefers concentrated portfolios. We estimate expected return models in global and emerging equity markets using a given stock selection model and generate statistically significant active returns from various portfolio construction techniques.

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In this study, we apply several portfolio construction and optimization techniques to global and emerging stock universes. We estimate expected return models in the global and emerging equity markets using a given stock selection model and generate statistically significant active returns from various portfolio construction techniques. In the first section, we introduce the reader to the risk and return trade-off analysis. In the second section, we examine the relationship of the (traditional) Markowitz mean-variance (MV) portfolio construction model with a fixed upper bound on security weights, and the Markowitz enhanced-index tracking (EIT) portfolio construction model in which security weights are an absolute deviation from the security weight in the index. We will refer to the absolute deviation from the benchmark weight enhanced index portfolio construction weight as the equal active weighting (EAW) portfolio construction model. In section three, we discuss portfolio construction and simulation, and present the empirical results. In section four, we offer conclusions and a summary.

We report that (1) mean-variance techniques continue to produce portfolios capable of generating excess returns above transactions costs and statistically significant asset selection, (2) optimization techniques minimizing expected tail loss are statistically significant in portfolio construction; and (3) global and emerging markets offer the potential for high returns relative to risk.

Introduction

Markowitz developed a portfolio construction model to achieve the maximum return for a given level of risk or the minimum risk for a given level of return [1, 2, 3, 4]. It has long been noted by Solnik [5, 6] that investors should diversify internationally rather than domestically and the number of securities is much larger than the U.S. market's securities. To better extend the portfolio construction methodology and techniques on U.S. market to the international market, we briefly review the applied U.S. and Global equity investment research in Guerard, Gultekin, and Xu [7] and Guerard, Markowitz, and Xu [8]. We test whether a mean-variance optimization technique using the portfolio variance as the relevant risk measure dominates the risk-return trade-off curve using a variation of the optimization model that emphasizes systematic (or market) risk and a mean-expected tail loss portfolio optimization techniques. A statistically-based Principal Components Analysis (PCA) model is used to estimate and monitor portfolio risk.

A measure of the trade-off between the portfolio expected return and risk (as measured by the portfolio standard deviation) is typically denoted by the Greek letter lambda (λ). Generally, the higher the lambda, the higher is the ratio of portfolio expected return to portfolio standard deviation. We assume that the portfolio manager seeks to maximize the portfolio geometric mean (GM) and Sharpe ratio (ShR) as put forth in Latane [9] and Markowitz [3, 10]. The reader is referred to Elton, Gruber, Brown, and Goetzmann [11] for a complete discussion of modern portfolio theory.

Summary and Findings

We report that the Markowitz Mean-Variance (MV) optimization technique, the Enhanced Index-Tracking (EIT or EAW) optimization technique, and tracking error at risk models are appropriate tools for portfolio construction for the USER and GLER data. Global portfolios dominate domestic portfolios with regard to the return-to-risk statistics for the 1999 – 2009 period. Moreover, the optimization techniques can be implemented in Emerging Markets. The Markowitz approach to portfolio construction and management is sixty years old and remains an integral tool of investment research.

Constructing Mean-Variance Efficient Portfolios

Portfolio construction and management, as formulated in Markowitz seeks to identify the efficient frontier, the point at which the portfolio return is maximized for a given level of risk, or equivalently, portfolio risk is minimized for a given level of portfolio return. The portfolio expected return, denoted by $E(R_\mathrm{P})$, is calculated by taking the sum of the security weight multiplied by their respective expected return:

$$E(R_\mathrm{P}) = \sum_{i=1}^{N} w_i E(R_i) \tag{1}$$

The portfolio standard deviation is the sum of the weighted securities covariances:

$$\sigma_\mathrm{p}^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \mathrm{w}_i w_j \sigma_{ij} \tag{2}$$

where N is the number of candidate securities, w_i is the weight for security i such that $\sum_{i=1}^{N} w_i = 1$ indicating that the portfolio is fully invested, and $E(R_i)$ is the expected return for security *i.*

The Markowitz framework measures risk as the portfolio standard deviation, a measure of dispersion or total risk. One seeks to minimize risk, as measured by the covariance matrix in the Markowitz framework, holding constant expected returns. The decision variables estimated in the Markowitz model are the security weights. The Markowitz model minimized the total risk, or variance, of the portfolio. Investors are compensated for bearing total risk.

We introduce two extensions to the mean-variance approach: an enhanced index tracking optimization (EIT) technique and a tracking error at risk optimization technique. Markowitz [4] rewrites the general portfolio construction model variance, V, to be minimized as:

$$\mathrm{V} = (\mathrm{X} - \mathrm{W})^{\mathrm{T}} C (X - W) \tag{3}$$

where $W^T = (W_1, \ldots, W_n)$ is the vector of weights of an index (or benchmark), X are the portfolio weights, and C *is the variance-covariance matrix* of security returns. Guerard, Takano, and Yamane [12] reported the efficiency of the EIT procedure is minimizing realized tracking errors.

One creates portfolios by allowing portfolio weights to differ from index weights by plus or minus 1%, up to 5%. Each portfolio denoted by EAW followed by a number indicating the percent. Obviously, one can use an infinite set of EAW variations. Guerard, Krauklis and Kumar [13] employed mean-variance and enhanced index tracking optimization techniques to test whether equal active weighting strategies of one, two, three, four , and five percent (weight deviations from the index, or benchmark, weights) outperform mean-variance strategies using four and seven percent maximum security weights. Guerard, Krauklis and Kumar [13] also reported that MV portfolios produced higher Information Ratios and Sharpe Ratios than EAW portfolios with weights less than EAW4. Thus, the traditional Markowitz Mean-Variance is (still) quite relevant in the world of business.

APT Multi-Factor Models in Business

The estimation of variance-covariance matrix has generated its own interest. Sharpe [14] introduced a single factor risk model and diagonalized the variance-covariance matrix by introducing a fictitious security, which simplifies the critical line algorithm proposed by Markowitz [2,3] for tracing out efficient frontier under general constraints and variance-covariance matrix.

Multi-Factor Risk Models evolved in the works of Rosenberg [15], Ross [16], and Ross and Roll [17]. The fundamentally-based domestic Barra risk model was developed in Rosenberg, Rosenberg and Marathe [18] and thoroughly discussed in Rudd and Clasing [19] and Grinhold and Kahn [20]. Barra attribution is used in this analysis to access stock selection statistical significance. The Barra model remains the industry standard for risk model. The total excess return for a multiple-factor model (MFM) in the Rosenberg methodology for security *j*, at time *t*, dropping the subscript *t* for time, may be written like this:

$$R_j = \sum_{k=1}^{K} \beta_{jk} \widetilde{f_k} + \widetilde{e_j} \tag{4}$$

The nonfactor, or asset-specific return on security *j*, is the residual risk of the security after removing the estimated impacts of the *K* factors. The term f_k is the rate of return on factor *k*. An extensive review of factor risk models can be found in Connor and Korajczyk [21].

Guerard [22] demonstrated the effectiveness of the Blin and Bender APT and Sungard APT systems in portfolio construction and management. The determination of security weights, the ws, in a portfolio is the primary calculation of the Markowitz portfolio management approach. The security weight is the proportion of the portfolio value invested in the individual j security. The portfolio weight of security j is calculated as

$$w_{(\mathrm{P})j} = \frac{MV_j}{MV_{\mathrm{P}}} \tag{5}$$

Where MV_j is the market value of security j and $w_{(\mathrm{P})j}$ is the portfolio market value.

The active weight of the security, $w_{(\mathrm{a})j}$ is calculated by subtracting the security weight in the (index) benchmark b, $w_{(\mathrm{b})j}$, from the security weight in the portfolio:

$$w_{(\mathrm{a})j} = w_{(\mathrm{P})j} - w_{(\mathrm{b})j} \tag{6}$$

Markowitz analysis and its efficient frontier minimize risk for a given level of return. Blin and Bender created APT, Advanced Portfolio Technologies, and its Analytics Guide [23, 24], which built upon the mathematical foundations of their APT system, published in Blin, Bender and Guerard [25]. Our review draws upon the APT Analytics Guide. Volatility can be decomposed into independent variance components, systematic and specific risk.

$$\sigma_{\mathrm{P}}^2 = \sigma_{\beta\mathrm{P}}^2 + \sigma_{\mathrm{SP}}^2 \tag{7}$$

where

σ_{P}^2 = total portfolio variance;

$\sigma_{\beta\mathrm{P}}^2$ = systematic portfolio volatility;

σ_{SP}^2 = specific portfolio volatility.

Tracking error is a measure of volatility applied to the active return of funds (portfolio) benchmark against an index. Portfolio tracking error is defined as:

$$\sigma_{\mathrm{te}} = \sqrt{Var(r_{\mathrm{P}} - r_{\mathrm{b}})} = \sqrt{E[(r_{\mathrm{P}} - r_{\mathrm{b}}) - E(r_{\mathrm{P}} - r_{\mathrm{b}})]^2} \tag{8}$$

where σ_{te} is the square root of the variance of annualized tracking error, and r_{P} and r_{b} are the actual (annual) portfolio return and benchmark return respectively. Systematic tracking error of a portfolio is a forecast of the portfolio active annual returns as a function of the securities returns associated with APT risk (factor) model components. The difference in APT portfolio returns versus a benchmark return can be written as:

$$d_{\mathrm{P,b}} = \left(\sum_{i=1}^{n_{\mathrm{P}}} w_{(\mathrm{P})i} r_{ti} - \sum_{j=1}^{n_{\mathrm{b}}} w_{(\mathrm{b})j} r_{tj}\right) \tag{9}$$

where n_{P} is the number of securities in the portfolio, and n_b is the number of securities in the portfolio benchmark. Although portfolios often contain stocks not in the benchmark, we can make $n_{\mathrm{P}} = n_{\mathrm{b}} = m$, by inserting zeros in the weights when appropriate. Let us define column vectors w_{P} and w_{b} for given portfolio and benchmark portfolios respectively. Then

$$d_{\mathrm{P,b}} = r_t \cdot (w_{\mathrm{P}} - w_{\mathrm{b}}) \tag{10}$$

Blin and Bender mimic the APT model in the Analytics Guide:

$$\sigma_{\mathrm{P,b}}^2 = Var\left(d_{\mathrm{P,b}}\right) = (w_{\mathrm{P}} - w_{\mathrm{b}})'(B'B + \Sigma)(w_{\mathrm{P}} - w_{\mathrm{b}}) \tag{11}$$

where B is the $k \times m$ matrix of factor loading and $\Sigma = e'e$, the $m \times m$ diagonal matrix of the specific risk loading. Thus the annualized APT calculated portfolio tracking error versus a benchmark is:

$$\sigma_{P,b} = \sqrt{52(w_{\mathrm{P}} - w_{\mathrm{b}})'(B'B + \Sigma)(w_{\mathrm{P}} - w_{\mathrm{b}})} \tag{12}$$

and

$$\sigma_{\mathrm{ste}} = \sqrt{52(w_{\mathrm{P}} - w_{\mathrm{b}})'\Sigma(w_{\mathrm{P}} - w_{\mathrm{b}})} \tag{13}$$

and $\sigma_{\mathrm{P,b}}^2 - \sigma_{\mathrm{ste}}^2$ is the systematic tracking variance of the portfolio and its square root is the systematic tracking error.

We can define the portfolio Value-at-Risk (VaR) as the probability that the value of the portfolio is going to decline, from its current value, V_0, by ate least the amount $V(\alpha, T)$ where T is the time horizon and α is a specified parameter, i.e. $\alpha = 0.05$, then

$$\mathrm{Prob}\big(V_T > V_0 - V(\alpha, T)\big) \geq 0.95 \text{ or}$$
$$\mathrm{Prob}\big(V_T < V_0 - V(\alpha, T)\big) \leq 0.05, if\ \alpha = 0.95 \quad (14)$$

The second case says that the probability that the value of the portfolio will decline by an amount $V(\alpha, T)$ with T holding period is at most 0.05.

Blin, Bender and Guerard [25] used a 20-factor beta model of covariances based on 3.5 years of weekly stock returns data. The Blin and Bender Arbitrage Pricing Theory (APT) model followed the Roll factor theory, but Blin and Bender estimated at least 20 orthogonal factors. The trade-off curves in Guerard [22] were created by varying lambda, a measure of risk-aversion, as a portfolio decision variable. As lambda rises, the expected return of the portfolio rises and the number securities in the portfolio declines.

A General Stock Selection Model for U.S., Global and Emerging Equity Markets

In 1991, Markowitz headed the Daiwa Securities Trust Global Portfolio Research Department (GPRD). The Markowitz team estimated stock selection models using Graham and Dodd [26] fundamental valuation variables, earnings, book value, cash flow and sales, relative variables, defined as the ratio of the absolute fundamental variable ratios divided by the 60-month averages of the fundamental variables. Bloch, Guerard, Markowitz, Todd, and Xu [27] reported a set of approximately 200 simulations of United States and Japanese equity models. Guerard, Gultekin, and Xu [7] extended a stock selection model originally developed and estimated in Bloch, Guerard, Markowitz, Todd, and Xu [27] by adding price momentum variable, taking the price at time t-1 divided by the price 12 months ago, t-12, denoted PM, and the consensus (I/B/E/S) analysts' earnings forecasts and analysts' revisions composite analysts' efficiency variable (CTEF) to the stock selection model. Guerard [22] used the CTEF variable that is composed of forecasted earnings yield, EP, revisions, EREV, and direction of revisions, EB, identified as breadth, as created in Guerard, Gultekin, and Stone [28]. Guerard also reported domestic (U.S.) evidence that the predicted earnings yield is incorporated into the stock price through the earnings yield risk index. Moreover, CTEF dominates the historic low price-to-earnings effect, or high earnings-to-price, EP. The reader is referred to Guerard [29] for a more detailed analysis of the USER Model. Fama and French [30, 31, 32, 33] presented evidence to support the BP and price momentum variables as anomalies. Guerard, Gultekin, and Xu [7] referred to the stock selection model as a United States Expected Returns (USER) Model. We can estimate an expanded stock selection model to use as an input of expected returns in an optimization analysis.

The stock selection model estimated in this study, denoted as USER is:

$$TR_{t+1} = a_0 + a_1 EP_t + a_2 BP_t + a_3 CP_t + a_4 SP_t + a_5 REP_t + a_6 \mathrm{RBP_t} + a_7 RCP_t + a_8 RSP_t + a_9 CTEF_t + a_{10} PM_t + e_t \tag{15}$$

where:

EP = [earnings per share]/[price per share] = earnings − price ratio;

BP = [book value per share]/[price per share] = book − price ratio;

CP = [cash flow per share]/[price per share] = cash flow − price ratio;

SP = [net sales per share]/[price per share] = sales − price ratio;

REP = [current EP ratio]/[average EP ratio over the past five years];

RBP = [current BP ratio]/[average BP ratio over the past five years];

RCP = [current CP ratio]/[average CP ratio over the past five years];

RSP = [current SP ratio]/[average SP ratio over the past five years];

CTEF = consensus earnings − per − share I/B/E/S forecast, revisions and breadth;

PM = Price Momentum;

and

e = randomly distributed error term.

The USER model is estimated using a weighted latent root regression (WLRR), analysis on equation (15) to identify variables statistically significant at the 10% level; uses the normalized coefficients as weights; and averages the variable weights over the past twelve months. The 12-

month smoothing is consistent with the four-quarter smoothing in Bloch, Guerard, Markowitz, Todd, and Xu [27]. While EP and BP variables are significant in explaining returns, the majority of the forecast performance is attributable to other model variables, namely the relative earnings-to-price, relative cash-to-price, relative sales-to-price, price momentum, and earnings forecast variables. The CTEF and PM variables accounted 44 percent of the weights in the USER Model.

The USER Model, using the Markowitz [2, 3] mean-variance optimization simulation with a 4 percent upper bound of a security weight, 35 basis point threshold weight, 8 percent monthly turnover and a lambda of 200, simulated over the January 1998 – December 2007 period, produced a managed portfolio return of 13.5%, asset selection of 9.72% (t-value of 3.66), and an Information Ratio of 1.21. The USER Model, using the mean-variance tracking error at risk (MVTaR) optimization simulation with a 4 percent upper bound of a security weight, 35 basis point threshold weight, 8 percent monthly turnover and a lambda of 200, simulated over the corresponding period, produced a managed portfolio return of 14.6%, asset selection of 11.38% (t-value of 3.66), and an Information Ratio of 1.23, see [7, page 74]. Both mean-variance techniques were highly statistically significant in producing asset selection from the USER Model.

Guerard, Rachev, and Shao [34] estimated a Global Model, GLER, using equation (24) and the FactSet database for global securities during the January 1999 – December 2011 period. In the world of business, one does not access academic databases annually, or even quarterly. Most industry analysis uses FactSet database and the Thomson Financial (I/B/E/S) earnings forecasting database. We estimated Equation (15) for all securities on the Thomson Financial and FactSet databases, some 46,550 firms in December 2011. We estimated the GLER Model upon the FactSet universe for the 1999-2011 period. The average estimated GLER weights were:

The Time-Average Value of GLER Estimated Coefficients:

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
.048	.069	.044	.047	.050	.032	.039	.086	.216	.257

The Guerard et al. [34] simulation produced an active portfolio return of 10.93% versus the Russell Global Growth Index and an Information Ratio of 1.22.

Comparative Portfolio Simulation Results with the USER and GLER Models and Applications to Emerging Markets

Deng and Min [35] tested the Guerard, Gultekin, and Xu [7] USER Model and the Global, GLER, Model of Guerard, Rachev, and Shao [34] during the 1999 – 2009 period. The portfolio returns of the USER model with APT MVTar and a lambda of 200 are shown in Table 1. We used the MVTaR optimization simulation with a 4 percent upper bound of a security weight, 35 basis point threshold weight, 8 percent monthly turnover, a lambda of 200, and constrained the number of stocks not to exceed 95 securities. Integer programming is used to constrain the number of stocks in the portfolios. For a given mean-variance tracking error at risk, MVTaR, model, the global model produces higher excess returns than the domestic model, with a smaller standard deviation. Moreover, the Information Ratio is 1.05 for the USER Model and 1.22 for the GLER Model. The global model has higher factor returns, primarily due to medium momentum, which is most effectiveness in the global markets.

Table 1: Tracking Error at Risk, TaR						
January 1999 - November 2009, Lambda = 200, 95 Stocks in Portfolio (Max)						
	Portfolio	Portfolio	Portfolio	Information	Specific	Factor
	Geometric	Standard	Excess	Ratio	Return	Return
APT Optimization Techniques	Mean	Deviation	Returns	(t-value)	(t-value)	(t-value)
Mean-Variance, MV, USER	8.68	22.36	8.68	1.05 (3.46)	6.83 (3.89)	1.84 (1.03)
Mean-Variance, MV, GLER	12.62	21.18	10.93	1.22 (3.83)	7.01 (3.31)	3.91 (2.07)
Axioma Attribution Factor Exposures						
	Medium					
	Momentum (t)	Value	Volatility	Growth		
Mean-Variance, MV, USER	.416 (3.33)	.325 (8.19)	.349 (-4.36)	.182 (3.64)		
Mean-Variance, MV, GLER	.487 (6.95)	.153 (5.44)	.501 (-4.01)	.392 (5.21)		

The USER and GLER Model portfolios have positive exposure to value and momentum and smaller stocks are purchased.

Global modeling for a “global growth specialist”, such as McKinley Capital Management, LLC, involves the use of larger weighting of momentum and forecasted earnings acceleration factors [36]. We refer to the forecasted earnings acceleration factor, approximated by CTEF, as E’. The global growth variables used in this analysis are:

PM121 = price momentum as $Price_{t-1}/Price_{t-12}$;
PM71 = price momentum as $Price_{t-1}/Price_{t-7}$;
FEP1 = one-year-ahead forecast earnings per share/price per share;
FEP2 = two-year-ahead forecast earnings per share/price per share;
RV1 = one-year-ahead forecast earnings per share monthly revision/price per share;
SIGMA = one-year daily return standard deviation;
RV2 = two-year-ahead forecast earnings per share monthly revision/price per share;
FGR1 = one-year-ahead forecast earnings per share monthly breadth;
FGR2 = two-year-ahead forecast earnings per share monthly breadth;
FEP1 = one-year-ahead forecast earnings per share/last year’s reported earnings per share;
FEP2 = two-year-ahead forecast earnings per share/ last year’s reported earnings per share;
CTEF = equally-weighted FEP1, FEP2, BR1, BR2, RV1, and RV2;
and MQ = .4*CTEF + .4*PM71 +.2*SIGMA.

We run tracking error at risk simulations with a 4 percent upper bound of a security weight, 35 basis point threshold weight, 8 percent monthly turnover, a lambda of 200, and an unconstrained the number of stocks, versus the MSCI All World Country Investible universe for the January 2000 – November 2013 period and show portfolio descriptive statistics using the MSCI Barra Attribution system in Table 2a for the mean-variance tracking error at risk estimates and for the enhanced index tracking, or equal active weighting, optimization techniques in Table 2b. The seven-month price momentum, PM71, beats the twelve-month price momentum variable, PM121, a result found in [36] and price momentum and E’, CTEF, dominate the global models. The McKinley Quant public model of momentum and E’, MQ, variable is less than either the PM71 or CTEF variable because of the standard deviation, SIGMA. The proprietary model, MQ, produces a higher portfolio return than the public MQ and hence a higher Information Ratio. The EAWTaR optimization technique is particularly helpful with the Proprietary MQ variable in global markets.

Table 2a: Mean-Variance Tracking Error at Risk					
Universe: All Country World Investible Growth					
January 2002 - June 2013, Lambda = 200					
	Annual	Annualized	Excess	Information	Tracking
Model	Returns	STD	Returns	Ratio	Error
EP	18.20	23.93	11.59	0.96	12.13
BP	8.13	27.38	1.52	0.08	18.25
CP	11.55	22.38	4.90	0.42	11.65
SP	15.63	23.38	9.02	0.68	13.29
PM71	15.03	22.09	8.43	0.73	11.58
PM121	11.38	23.50	4.78	0.38	12.56
Alpha	10.18	23.06	3.57	0.28	12.63
CTEF	14.68	23.19	8.07	0.67	12.12
SIGMA	9.90	12.65	3.30	0.34	9.6
MQ	12.42	17.68	5.82	0.68	7.66
Proprietary MQ	17.62	19.01	11.02	1.29	8.57
Benchmark	6.60	12.73			

Table 2b: Enhanced Index Tracking Error at Risk Portfolio Optimization					
Universe: All Country World Investible Growth					
January 2002 - June 2013, Lambda = 200					
	Annual	Annualized	Excess	Information	Tracking
Model	Returns	STD	Returns	Ratio	Error
EP	15.52	24.09	9.41	0.86	10.96
PM71	15.16	23.84	9.05	0.81	11.16
Alpha	10.49	23.26	7.14	0.36	11.2
CTEF	13.20	22.52	7.09	0.71	8.05
MQ	16.60	18.33	7.49	0.93	12.23
Proprietary MQ	18.66	18.73	12.06	1.48	8.17

One can ask why we use a lambda of 200 for the global GLER Model is used at McKinley Capital. In Table 3, we estimate the MVTaR Proprietary MQ model for various lambdas, creating an Efficient Frontier. We use integer programming to limit securities in portfolios to 95 and use our standard optimization conditions: (1) 8 percent monthly turnover; (2) 4 % maximum security weight; (3) 35 basis point threshold weight; and (4) an 8% systematic tracking error upper bound. [1]

[1] An 8% systematic tracking error is consistent with promised tracking errors by McKinley Capital.

Table 3: Mean-Variance Tracking Error at Risk						
Universe: All Country World Investible Growth						
Portfolios Constrained to 95 stocks, Eight Percent Systematic Tracking Error						
January 2002 - June 2013						
	Annual	Annualized	Excess	Information	Tracking	
Lambda	Returns	STD	Returns	Ratio	Error	
1	8.11	17.76	1.51	0.27	5.50	
10	12.02	18.69	5.42	0.78	6.96	
50	15.61	18.44	9.01	1.00	7.50	
100	16.77	18.37	10.70	1.28	7.93	
200	17.41	18.25	10.80	1.32	8.18	
500	18.30	18.42	11.69	1.31	8.96	

One can perform the identical global analysis on the MSCI Emerging Markets Growth index constituents for the identical period and find similar results, see Table 4. Momentum and E' dominate the Emerging Markets Index constituents during the 2000 – 2013 optimization period. There is greater diversification between momentum and E' in emerging markets than in Global markets as one sees a higher MQ return than the respective PM71, CTEF, and SIGMA variables. In January 2000, Emerging Markets composed 6.21% of the market capitalization of the MSCI All Country World Growth (ACWG) Index, up substantially from its 1.65% weight in January 1997 when the ACWG was launched. An outstanding resource on Emerging Markets history is the MSCI Barra publication [37].

Table 4: APT Mean - Variance Estimated Tracking Error at Risk Models
Universe: Emerging Markets Constituents
January 2002 -June 2013
Simulation Conditions: Lambda =200; monthly turnover =8%; maximum security weight =4%;
35 basis point threshold investment; equal active weight bounds are + / - 2% of index.

Model	Annualized Returns	Annualized Standard Deviation	Excess Returns	Information Ratio	Sharpe Ratio	Tracking Error
EP	18.20	23.93	11.59	0.96	0.70	12.13
BP	8.13	27.38	1.52	0.08	0.24	18.25
CP	11.55	22.38	4.95	0.04	0.45	11.65
SP	15.63	23.37	9.02	0.68	0.60	13.29
PM71	15.03	22.09	4.78	0.68	0.60	11.68
PM12	11.38	23.50	8.43	0.73	0.42	12.56
Alpha	10.18	23.08	3.57	0.28	0.37	12.63
CTEF	14.68	23.19	8.07	0.67	0.57	17.02
SIGMA12	9.90	12.65	3.30	0.34	0.59	9.60
MQ	12.42	18.52	5.82	0.68	0.59	8.50
Proprietary MQ	17.62	19.01	6.76	1.29	0.85	7.66
Benchmark	6.60	16.37				

Guerard et al. [34] reported an Axioma Fundamental Risk Model in which Medium-term Momentum, Value, and Growth style factors contributed statistically significant returns to the Global GLER portfolio. In Table 5, we report how the CTEF variable produces an eight percent active return in Emerging Markets over the January 2000 – November 2013 period. The CTEF produces 239 basis points of specific returns, which is statistically significant (t= 2.03). The CTEF has large exposures to the Medium-term Momentum, Value, and Growth style factors which produce statistically significant returns and 369 basis points of style returns.

Table 5: Factor Attribution: Factor Contributions

Portfolio: MVTaR_CTEF-200_EM
Benchmark: Russell Global
Period: 2000-01-31 to 2013-11-29 (Monthly)
Risk Model: WW21AxiomaMH
Base Currency: USD

Returns Scaling: Annualized (Geometric)
Risk Type: Predicted Risk

Source of Return	Contribution	Avg Exposure	Hit Rate	Risk	IR	T-Stat
Portfolio	**12.85%**			**18.65%**		
Benchmark	**4.85%**			**19.46%**		
Active	**8.00%**			**6.90%**	**1.16**	**4.31**
Specific Return	**2.39%**			**4.37%**	**0.55**	**2.03**
Factor Contribution	**5.61%**			**5.33%**	**1.05**	**3.91**
Style	**3.69%**			**2.52%**	**1.47**	**5.46**
Exchange Rate Sensitivity	-0.04%	-0.0036	43.37%	0.20%	-0.20	-0.75
Growth	0.33%	0.2058	62.05%	0.27%	1.24	4.61
Leverage	-0.11%	0.1371	48.19%	0.25%	-0.45	-1.69
Liquidity	0.22%	0.1689	56.02%	0.39%	0.56	2.07
Medium-Term Momentum	1.98%	0.3060	69.28%	1.04%	1.91	7.11
Short-Term Momentum	-0.29%	0.0430	46.39%	0.72%	-0.40	-1.48
Size	0.42%	-0.2324	54.82%	1.86%	0.23	0.85
Value	1.99%	0.5863	67.47%	0.90%	2.20	8.19
Volatility	-0.82%	0.1251	42.17%	1.18%	-0.69	-2.57
Country	**1.54%**			**2.88%**	**0.53**	**1.99**
Industry	**0.26%**			**2.24%**	**0.11**	**0.43**
Currency	**0.34%**			**1.40%**	**0.24**	**0.90**
Local	**0.00%**			**0.01%**	**-0.02**	**-0.07**
Market	**-0.22%**			**2.08%**	**-0.11**	**-0.39**

By contrast in Emerging Markets, in Table 6, we report that the Proprietary MQ variable produces its 16.40 percent active return in Emerging Markets over the January 2000 – November 2013 period. The Proprietary MQ produces 1266 basis points of specific returns, which is highly statistically significant (t= 8.80). The Proprietary MQ variable has large exposures to the Medium-term Momentum and Growth style factors which produce statistically significant returns. Momentum and Growth style returns are offset by its negative exposures to Value and Liquidity, such that the Proprietary MQ variable produces -22 basis points of style returns. The most important result is that the Proprietary MQ variable produces excess returns primarily due to stock selection (labeled as specific return).

Table 6: Factor Attribution: Factor Contributions

Portfolio: EAW_NoRCProprietaryMQ_EM
Benchmark: Russell Global
Period: 2000-01-31 to 2013-11-29 (Monthly)
Risk Model: WW21AxiomaMH
Base Currency: USD

Returns Scaling: Annualized (Geometric)
Risk Type: Predicted Risk

Source of Return	Contribution	Avg Exposure	Hit Rate	Risk	IR	T-Stat
Portfolio	**21.25%**			**20.46%**		
Benchmark	**4.85%**			**19.46%**		
Active	**16.40%**			**12.02%**	**1.36**	**5.07**
Factor Contribution	**3.75%**			**10.77%**	**0.35**	**1.29**
Style	**-0.22%**			**3.97%**	**-0.06**	**-0.21**
Exchange Rate Sensitivity	0.05%	-0.0302	51.81%	0.21%	0.23	0.85
Growth	0.99%	0.4151	66.87%	0.50%	1.99	7.40
Leverage	0.13%	-0.0825	57.23%	0.15%	0.87	3.24
Liquidity	-0.88%	-0.3663	42.17%	0.90%	-0.99	-3.67
Medium-Term Momentum	3.74%	0.7449	69.28%	2.55%	1.47	5.46
Short-Term Momentum	0.20%	0.0682	44.58%	1.06%	0.19	0.70
Size	0.41%	-0.2483	54.82%	1.39%	0.29	1.10
Value	-0.91%	-0.2436	35.54%	0.44%	-2.05	-7.63
Volatility	-3.95%	0.2892	37.35%	2.15%	-1.83	-6.82
Country	**2.83%**			**8.71%**	**0.33**	**1.21**
Industry	**-0.21%**			**2.54%**	**-0.08**	**-0.31**
Currency	**1.39%**			**3.64%**	**0.38**	**1.42**
Local	**0.05%**			**0.32%**	**0.16**	**0.61**
Market	**-0.10%**			**2.24%**	**-0.05**	**-0.17**
Specific Return	**12.66%**			**5.35%**	**2.37**	**8.80**

In Table 7, we analyze the Emerging Markets strategy for the June 2006 – November 2013 Period to examine the relationship between the publication of the McKinley Capital Management, LLC, MCM, white paper [38] and its post-publication performance. We use the MSCI Emerging Markets Growth Index was the benchmark for the Emerging Markets portfolio in Table 7.

Table 7: EM Portfolio, 6/2006 -12/2013
ATTRIBUTION REPORT
Annualized Contributions To Total Return

Source of Return	Contribution (% Return)	Risk (% Std Dev)	Info Ratio	T-Stat
1 Risk Free	1.19			
2 Total Benchmark	6.23	25.53		
3 Currency Selection	-0.51	1.73	-0.27	-0.73
4 Cash-Equity Policy	0.00	0.00		
5 Risk Indices	3.10	3.04	0.82	2.26
6 Industries	-0.87	1.52	-0.53	-1.45
7 Countries	1.35	3.49	0.33	0.90
8 World Equity	0.00	0.00		
9 Asset Selection	3.37	3.70	1.00	2.74
10 Active Equity [5+6+7+8+9]	6.95	6.24	1.05	2.88
11 Trading				
12 Transaction Cost	-4.05			
13 Total Active [3+4+10+11+12]	2.51	6.53	0.39	1.06
14 Total Managed [2+13]	8.74	25.41		

ATTRIBUTION REPORT
Annualized Contributions To Risk Index Return

Source of Return	Average Active Exposure	Contribution (% Return)			Total		
		Average [1]	Variation [2]	Total [1+2]	Risk (% Std Dev)	Info Ratio	T-Stat
Momentum	0.52	1.09	1.27	2.37	2.07	1.04	2.85
Volatility	-0.17	0.42	-0.51	-0.09	2.00	-0.22	-0.61
Value	0.20	0.60	0.11	0.72	0.41	1.69	4.63
Size	-0.13	-0.06	0.09	0.03	0.31	0.05	0.14
Size Nonlinearity	0.10	0.15	-0.01	0.14	0.16	0.82	2.25
Growth	0.04	0.01	0.02	0.03	0.11	0.27	0.74
Liquidity	-0.05	0.07	-0.04	0.04	0.15	0.14	0.38
Financial Leverage	0.06	-0.07	-0.06	-0.13	0.15	-0.64	-1.76
Total				3.10	3.04	0.82	2.26

The Emerging Markets strategy offers positive and statistically significant factor returns because Momentum has added 237 basis points annually (t-statistic =2.85) to the Emerging Markets portfolio. Momentum dominated the factor returns. The Proprietary MQ model has produced asset selection of 337 basis points for the June 2006 – November 2013 period. The active equity returns of the Emerging Markets portfolio were 695 basis points and were highly statistically significant (t-statistic = 2.88). The Momentum, asset selection, and excess returns in the MCM white paper [38] were maintained in the post-publication period.

We use a third period of December 2009 – November 2013, because McKinley Capital Management, LLC, launched an Emerging Markets portfolio in February 2011.[2] By December 2009, the Emerging Markets stocks composed 10.44% of the ACWG market capitalizations. Emerging Markets grew from 10.44% in December 2009 to 11.07% in November 2013. The portfolio simulations of the latter period are very important with the launch of an Emerging Markets fund, see Table 8.

Table 8: ATTRIBUTION REPORT of the MQ Model, December 2009 - November 2013
Annualized Contributions To Total Return

Source of Return	Contribution (% Return)	Risk (% Std Dev)	Info Ratio	T-Stat
1 Risk Free	0.08			
2 Total Benchmark	4.08	21.87		
3 Currency Selection	0.77	1.30	0.52	1.03
4 Cash-Equity Policy	0.00	0.00		
5 Risk Indices	4.48	2.19	1.87	3.74
6 Industries	-1.54	1.30	-1.00	-2.01
7 Countries	2.43	2.41	1.00	2.00
8 World Equity	0.00	0.00		
9 Asset Selection	0.78	2.96	0.29	0.58
10 Active Equity [5+6+7+8+9]	6.15	4.38	1.38	2.76
11 Trading				
12 Transaction Cost	-3.66			
13 Total Active [3+4+10+11+12]	3.36	4.59	0.77	1.53
14 Total Managed [2+13]	7.44	22.50		

The average active Momentum exposure of the MQ Model was 0.54, which produced an average annual contribution of return of 345 basis points. The Information Ratio on the Momentum exposure was 1.43 and its t-statistic was 2.85. The Momentum exposure of the Emerging

[2] We use the 12/2009 – 11/2013 period because the McKinley Capital Management, LLC, (MCM) Emerging Markets portfolio was launched in March 2011. The portfolio simulated excess returns in Table 5 have been realized, and exceeded by over 200 basis points due to the MCM Qualitative research process.

Markets portfolio was highly statistically significant in generating risk index returns during the December 2009 – November 2013 period.

The USER, GLER, and MQ portfolios and the corresponding attribution analyses report statistically significant active returns based on specific asset selection. The mean-variance, equal active weighting, and tracking error at risk optimization techniques produce portfolios consistent with Geometric Mean and Information Ratio maximization.

Conclusions

We addressed several issues in portfolio construction and management with USER, GLER, and MQ data. First, we report that the Markowitz Mean-Variance (MV) optimization technique, the Enhanced Index-Tracking optimization technique, and tracking error at risk models are appropriate for USER, GLER, and MQ data. Global portfolios dominate domestic portfolios with regard to the return-to-risk statistics for the 1999 – 2009 period. The portfolios produce significant factor returns, through the Momentum exposure, and asset selection, driven by the forecasted earnings acceleration variable, E'. Optimization techniques can be applied to Emerging Markets, with promising and statistically significant initial results.

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