Monthly Beta Forecasting with Low, Medium and High Frequency Stock Returns

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Abstract

Generating one-month-ahead systematic (beta) risk forecasts is common place in financial management. This paper evaluates the accuracy of these beta forecasts in three return measurement settings; monthly, daily and 30 minutes. It is found that the popular Fama-MacBeth beta from 5 years of monthly returns generates the most accurate beta forecast among estimators based on monthly returns. A realized beta estimator from daily returns over the prior year, generates the most accurate beta forecast among estimators based on daily returns. A realized beta estimator from 30 minute returns over the prior 2 months, generates the most accurate beta forecast among estimators based on 30 minute returns. In environments where low, medium and high frequency returns are accurately available, beta forecasting with low frequency returns are the least accurate and beta forecasting with high frequency returns are the most accurate. The improvements in precision of the beta forecasts are demonstrated in portfolio optimization for a targeted beta exposure.

KEY WORDS: CAPM, portfolio optimization, systematic risk, time-series modeling.

JEL: C53, G17.

1 Introduction

Forecasting systematic risk (beta) has played an important role in financial management since the development of the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), where beta is defined as the ratio of a security's return covariance with the market return to the variance of the market return. Globally there is billions of dollars in assets under management with targeted beta exposure. The most common beta forecasting approach is that of Fama and MacBeth (1973) which uses monthly returns over the prior 5 years to compute this ratio. The popularity of this approach is due to historically there often being ready availability of monthly returns, rather than a strong econometric justification. However, in recent years the tremendous growth in availability of financial data has led to accurate higher frequency stock returns to be more accessable to forecasters of beta.

In response to this growth in availability of quality higher frequency financial data, the literature in financial econometrics has developed with new estimators and evaluation criteria for higher frequency data. Most notable has been the development of the realized volatility and realized beta literature. The realized volatility literature was initiated by Andersen and Bollerslev (1998) and Andersen et al. (2001a, 2001b and 2003), while the realized beta literature was initiated by Barndorff-Nielsen and Shepherd (2004) and Andersen et al. (2005 and 2006). Beta forecasting studies utilizing realized betas have been conducted in Hooper et al. (2008), Papageorgiou et al. (2010) and Reeves and Wu (2013) for the one-quarter-ahead horizon. For longer horizons, Chang et al. (2012) conduct forecast evaluations with realized betas for the 6-month, 1-year and 2-year horizons.

In this paper, forecasting beta risk for one-month-ahead is analyzed. The one-month-ahead forecast horizon is chosen due to its widespread use in the financial management industry, in particular in portfolio management where monthly forecasts of beta play an important role in portfolio construction for a targeted beta exposure. Forecasting beta with low, medium and high frequency stock returns are considered, corresponding to monthly, daily and 30 minute returns. Beta estimators from monthly stock returns are analyzed as for illiquid stocks, monthly returns are more reliable than higher frequency returns. For relatively liquid stocks, daily returns can be accurately measured and this paper demonstrates large improvements in beta estimators from daily stock returns, relative to beta estimators from monthly returns. Beta estimators from higher frequency (30 minute) stock returns are also analyzed as for very liquid stocks such as stocks currently in the S&P500 index, returns can be accurately measured at this frequency. Models evaluated include constant beta models, autoregressive models of realized beta and mixed-data sampling (MIDAS) models.

Ghysels (1998) with monthly stock returns finds constant beta models to be more accurate in forecasting beta, relative to time-varying beta models. The results of this current study find that for constant beta models estimated with monthly stock returns, the highest forecast accuracy comes from an estimation period of 60 months, following Fama and MacBeth (1973). In the setting of daily stock returns, this study finds that a constant beta model estimated over the prior year, delivers the most accuracy in forecasting beta for one-month-ahead. In the setting of 30 minute stock returns, a constant beta model estimated over the prior two months, delivers the most accuracy and provides better performance relative to autoregressive models of realized beta, initially studied by Andersen et al. (2006), Ghysels and Jacquier (2006) and Hooper et al. (2008). In environments where low, medium and high frequency returns are accurately available, beta forecasting with low frequency returns are the least accurate and beta forecasting with high frequency returns are the most accurate.

Both statistically and economically significant differences are demonstrated between the beta forecasts. Statistical testing is conducted with the Diebold and Mariano (1995) test, while economic testing is through stock ranking on beta forecasts and in constructing optimal portfolios for a target beta exposure. It is found that the approaches with lower beta forecast error, typically result in superior performance in constructing optimal portfolios, with the constant beta model estimated over the prior two months of 30 minute returns producing the best results overall.

This paper is organized as follows. Section 2 discusses the construction and justification of realized betas and section 3 describes the data. Section 4 evaluates a variety of beta forecasting approaches. Section 5 analyzes the beta forecasting approaches in portfolio optimization and section 6 concludes the paper.

2 Realized Beta Measurement

Following Barndorff-Nielsen and Shephard (2004) and Andersen et al. (2006) we assume the $N \times 1$ vector of security log price's $\boldsymbol{p}(t)$, follows a multivariate continuous-time stochastic volatility diffusion,

$$d\boldsymbol{p}(t) = \boldsymbol{\mu}(t)dt + \boldsymbol{\theta}_t d\boldsymbol{W}(t) \tag{1}$$

where $\boldsymbol{W}(t)$ is standard N-dimensional Brownian motion, $\boldsymbol{\omega}_t = \boldsymbol{\theta}_t \boldsymbol{\theta}_t^{'}$ is the instantaneous covariance matrix and $\boldsymbol{\mu}(t)$ is the N-dimensional instantaneous drift. Both $\boldsymbol{\omega}(t)$ and $\boldsymbol{\mu}(t)$ are strictly stationary and jointly independent of $\boldsymbol{W}(t)$. Let the i^{th} element of $\boldsymbol{p}(t)$ contain the log price of the i^{th} individual stock and the N^{th} element of $\boldsymbol{p}(t)$ contain the log price of the market. Suppose the process is sampled S times per period on an equally spaced grid and define the $\delta = 1/S$ period return as $\boldsymbol{r}_{t,j} = \boldsymbol{p}(t + j\delta) - \boldsymbol{p}(t + (j-1)\delta)$, $j = 1, 2, \ldots, S$. The realized beta of a security i, can be defined as the ratio of the realized covariance of security i and the market index N to the realized variance of the market index N, expressed as:

$$\beta_{i,t+1} = \frac{\sum_{j=1}^{S} r_{i,t,j} r_{N,t,j}}{\sum_{j=1}^{S} r_{N,t,j}^2} \tag{2}$$

which is a consistent estimator of the true underlying integrated beta,

$$\frac{\int_t^{t+1} \boldsymbol{\omega}_{iN}(\tau) d\tau}{\int_t^{t+1} \boldsymbol{\omega}_{NN}(\tau) d\tau} \tag{3}$$

almost surely for all t as $S \to \infty$. See Barndorff-Nielsen and Shephard (2004) for details.

3 Data

In this study, betas are analyzed for stocks trading in the Dow Jones Industrial Average Index (DJIA). Low, medium and high frequency stock returns are available for these stocks due to their high liquidity, over the entire sample period. Our study covers the period from 2nd Jan 1998 to 31st Jul 2009 which includes 24 stocks, listed in table 1. The market index is the DJIA. Initially the entire 30 companies of the DJIA were considered, however, due to incompleteness of data, 6 companies are excluded from the sample. High frequency (30 minute) stock returns are sourced from Price-Data (http://www.grainmarketresearch.com/). Daily and monthly stock returns (adjusted for dividends and stock splits) are sourced from CRSP (http://www.crsp.com/).

4 Forecast Evaluation

The primary forecasting approach considered for monthly one-step-ahead beta forecasts is from constant beta models. Two types of constant models are utilized; the Fama and MacBeth (1973) regression and the Barndorff-Nielsen and Shephard (2004) realized beta. In addition, when appropriate, autoregressive models of realized beta are considered following Andersen et al. (2006) and Hooper et al. (2008) and also the mixed-data sampling (MIDAS) models introduced by Ghysels et al. (2005 and 2006).

Constant beta models have been the dominant forecasting approach for beta since the 1970's. The Fama and MacBeth (1973) beta is still the most widely used approach. Ghysels (1998) with monthly stock returns demonstrates the dominance of constant beta models over time-varying beta models. More recently, Reeves and Wu (2013) for quarterly beta forecasting demonstrate a constant beta model dominating the autoregressive models of realized beta studied in Andersen et al. (2006) and Hooper et al. (2008). Continuing this research, this paper focuses on monthly beta forecasting, with low, medium and high frequency stock returns.

The Fama and MacBeth (1973) regression model for β_i is:

$$r_{i,t} = \alpha_i + \beta_i r_{m,t} + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim iid(0, \sigma^2) \;\; t = 1, 2, \ldots, n, \tag{4}$$

where $r_{i,t}$ and $r_{m,t}$ are the time t security i return and market return, respectively, measured at the monthly frequency, and the one-month-ahead β_i forecast is computed from running the regression over the previous n months. In our study the values of n are 24, 36, 48, 60, 72 and 80. These Fama and MacBeth forecasts are denoted as 24M(Monthly), 36M(Monthly), 48M(Monthly), 60M(Monthly), 72M(Monthly) and 80M(Monthly).

The realized beta forecast is computed from equation 2 from returns over the prior period. With daily returns this period is 1, 2, 3, 6, 12, 18, 24 and 48 months and is denoted by 1M(Daily), 2M(Daily), ... , 48M(Daily). With 30 minute returns this prior period is 1, 2, 3, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22 and 24 months and is denoted by 1M(30m), 2M(30m), ... , 24M(30m).

In the setting of 30 minute returns we also consider autoregressive models (AR(p)) of realized beta. These are not considered in the daily and monthly return measurement settings as there are insufficient return observations to compute a monthly realized beta for autoregressive modeling. With 30 minute returns there are approximately 280 observations per month. These realized betas are modeled with the following autoregressive specification for for $\beta_{i,t}$:

$$\beta_{i,t} = \phi_0 + \sum_{j=1}^{p} \phi_j \beta_{i,t-j} + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim iid(0, \sigma^2) \quad t = 1, 2, \ldots, n, \tag{5}$$

and the one-month-ahead forecast is based on estimation over the prior n months, for n = 24, 48, 72 and 100.

The MIDAS approach of Ghysels et al. (2005 and 2006) allows the estimation using data at different frequencies. In our framework, this approach allows us to forecast betas measured at lower frequencies using those measured at higher frequencies. Specifically, we use weekly realized betas to forecast monthly ones. Following Ghysels and Jacquier

(2006), the MIDAS regression in our paper can be formulated as follows:

$$\beta_{i,t} = \alpha_i + \phi_i \sum_{\kappa=1}^{\kappa max} B(\kappa, \theta)\hat{\beta}_{i,t-\kappa/week} + \epsilon_{i,t} \tag{6}$$

where the notation $t - \kappa/week$ lag operates according to the weekly sampling frequency. $\beta_{i,t}$ is the monthly realized beta as before and the regressors, $\hat{\beta}_{i,t-\kappa/week}, \kappa = 1 \ldots, \kappa_{max}$ are the weekly realized betas measured based on 30 minute returns within the week. κ_{max} is the maximum number of lags used in the MIDAS regression and we consider $\kappa_{max} = 2, 4, 8, 12, 16, 20$. $B(\kappa, \theta)$ is a function of parameters θ that need to be estimated. As one considers more lags, the number of parameters might increase, causing a curse of dimensionality. One of the advantages of the MIDAS is approach is that it solves the curse of dimensionality problem by considering a tightly parameterized function of θ and, thus, substantially decreasing the number of parameters to be estimated. The parameterization scheme that we utilize is the "Exponential Almon Lag" with a lag order of two, i.e. $B(\kappa, \theta) = \exp \theta_1\kappa + \theta_2\kappa^2 / \sum_{\kappa}^{\kappa_{max}} \exp \theta_1\kappa + \theta_2\kappa^2$. However, the parameter estimates are known to be sensitive to the initial starting values. To overcome this issue, we search over potential starting values for the parameters based on simulated annealing. Estimation is over data commencing from the start of the sample period.

One-month-ahead forecasts of beta for each of our stocks are evaluated by two alternative measures; Mean Squared Error (MSE) and Mean Absolute Error (MAE). The MSE and MAE are calculated as follows:

$$MSE = \frac{1}{m} \sum_{j=1}^{m} (\widetilde{\beta_{i,j}} - \widehat{\beta_{i,j}})^2 \tag{7}$$

$$MAE = \frac{1}{m}\sum_{j=1}^{m}|\widetilde{\beta_{i,j}} - \widehat{\beta_{i,j}}| \tag{8}$$

where m is the total number of forecasting periods, $\widehat{\beta_{i,j}}$ is the forecasted i_{th} stock's j_{th} period beta and $\widetilde{\beta_{i,j}}$ is the monthly realized beta computed from 30 minute returns for the i_{th} stock in the j_{th} period. The forecast evaluation period is over 40 months from May 2006 to July 2009 and the $\widetilde{\beta_{i,j}}$ are displayed in figure 1.

The MSE and MAE for each stock over a range of models are displayed in tables 2 and 3, with the lowest forecast error for each stock in bold. The 2m(30m) model produces the most accurate forecasts, followed by the 4m(30m). Over our 24 stocks, the 2m(30m) and 4m(30m) models produce the lowest MAE for 10 and 5 stocks, respectively. And similar results are found when the forecast evaluation loss function is MSE.

Tables 4 and 5 display the MSE and MAE averaged over all stocks, for each forecasting approach. The 2m(30m) has the lowest MAE, followed by the AR(3) with n=48 and the 1m(30m). A similar ordering occurs with MSE, though the AR(3) with n=48 has a slightly lower MSE than the 2m(30m). The MIDAS models perform relatively poorly when compared to the other approaches that utilize 30 minute returns. The MAE of the best MIDAS model is 0.2136, whereas the MAE of the 2m(30m) model 0.1681.

When only models using daily returns are considered, the 12M(Daily) produces the most accurate forecasts, delivering the lowest MSE and MAE. When only models using monthly returns are considered, the 60M(Monthly) produces the most forecast accuracy, delivering the lowest MSE and MAE. When both daily and monthly returns are available, the best forecaster from using monthly returns, generates a MSE over double that of the best forecaster from using daily returns. i.e. the MSE of the 60M(Monthly) is 0.1563 versus a MSE of 0.0741 for the 12M(Daily) model.

In addition, the Diebold and Mariano (1995) test (DM test) is used to examine if a given beta forecast is statistically different than an alternate forecast. The DM test is a simple and model free test of equal predictive accuracy, i.e. equal expected loss. In essence, it is simply an asymptotic z-test of the hypothesis that the loss functions evaluated at errors from two forecasts have the same mean. Specifically, let ε_t^1 and ε_t^2 for $t = 1, \ldots, T$ denote the time series of forecast errors for the out-of-sample period of T observations from two forecasting models. Let $L(\varepsilon_t^i)$ denote the loss function, such as squared error loss, i.e. $L(\varepsilon_t^i) = (\varepsilon_t^i)^2$, or absolute error loss, i.e. $L(\varepsilon_t^i) = |\varepsilon_t^i|$. The DM test is based on the loss differential $d_t = L(\varepsilon_t^1) - L(\varepsilon_t^2)$. Thus, the null of equal predictive accuracy can be expressed as $H_0 : E[d_t] = 0$ and can be tested against one- or two-sided alternatives. The DM test statistic is $S = \bar{d}/(\widehat{avar}(\bar{d}))^{1/2}$ where $(\widehat{avar}(\bar{d}))^{1/2}$ is a consistent estimate of the asymptotic variance of $\bar{d}$. The DM test statistic has an asymptotic standard normal distribution under the null of equal predictive accuracy. In this paper, we consider both squared and absolute loss functions for the DM tests and compare the better performing forecasting models from the different return measurement frequencies, i.e. 2M(30m), 12M(Daily), 60M(Monthly) and the AR(3) with n=48, given that these models have the stronger forecasting performance based on their MSE and MAE results. We use the simple sample variance as an estimate of the asymptotic variance of the loss differential.

We run the test for 4 models and 6 different combinations over the evaluation period July 2002 to July 2009, examining if a given forecast is statistically different to an alternate forecast at the 5 percent level. The DM test results are reported in tables 6 and 7. Table 6 results are based on squared forecasting errors and table 7 results are based on the absolute value of forecasting errors. The first column in table 6 shows that for a number of stocks, the 2M(30m) model is statistically different to that of the 12M(Daily) model, for example, for company MMM, IBM, MCD, KO and JPM, the 2M(30m) model has superior beta forecast performance than the 12M(Daily) model at the 5 percent significant level. However for the other stocks, the two models have statistical insignificant differences.

The next two columns compare the 60M(Monthly) model with 2M(30m) and 12M(Daily) models, and illustrate that the 60M(Monthly) model is under-performing. For 19 out of 24 companies, the 2M(30m) model does better and similar results are also found in the 12M(Daily) case. In the last three columns, we compare the AR(3) with n=48 model with the 2M(30m), 12M(Daily) and 60M(Monthly) models. For about half of the stocks there are statistically significant differences between the 2M(30m) model and the AR(3) with n=48 model, and also between the AR(3) with n=48 model and the 12M(Daily) model. For the majority of stocks, the AR(3) with n=48 model is statistically different to the 60M(Monthly) model. Similar results are found in table 7 with the absolute value of forecasting errors.

In table 8 we rank stocks by their 2M(30m), 12M(Daily), AR(3) with n=48 and 60M(Monthly) beta forecasts. As this is common practice in investment management, it provides an economic interpretation to the variability of the beta forecasts. Most notable is the substantial difference in the 60M(Monthly) rank, relative to the other methods. For example, the Exxon Mobil Corporation beta forecast for July 2009 is 0.5151 from the 60M(Monthly) placing it as second ranked, whereas the beta forecast is 1.0962 from the 12M(Daily) placing it as fifteenth ranked.

5 Portfolio Optimization

In this section, we consider an application of the beta model in asset allocation and portfolio optimization. Since portfolio systematic risk is measured by the portfolio beta, and the portfolio beta is the weighted average of individual stocks' beta in that portfolio, accurate beta measurement is essential to the evaluation of portfolio systematic risk.

There is considerable evidence that superior returns to investment performance are elusive and in practice, managers are often evaluated relative to a certain benchmark, such as a market index. Therefore one of their primary objectives is to minimize the portfolio's volatility, while maintaining the same risk as the market. In the following, we consider a professional investment manager who is trying to construct a portfolio with beta equal to one, and minimizing the volatility of her portfolio at the same time. We then evaluate which beta forecasting approach generates the optimal portfolio, as in Ghysels and Jacquier (2006).

Let $\boldsymbol{R}_t$ denote the 1×24 vector of individual stock returns on day t. On the first day t of every month m, the manager will use the return series to estimate covariances and generate a covariance matrix forecast $\boldsymbol{\Omega}_m$ for month m. After that, to construct the minimum tracking error variance portfolio in month m, the manager simply applies the following weights with the 24 DJIA stocks:

$$\mathbf{W} = \frac{\boldsymbol{\Omega}^{-1}[\boldsymbol{\beta}(\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1} - \boldsymbol{\beta}'\boldsymbol{\Omega}^{-1}\mathbf{1}) + \mathbf{1}(\boldsymbol{\beta}'\boldsymbol{\Omega}^{-1}\boldsymbol{\beta} - \boldsymbol{\beta}'\boldsymbol{\Omega}^{-1}\mathbf{1})]}{\boldsymbol{\beta}'\boldsymbol{\Omega}^{-1}\boldsymbol{\beta}\mathbf{1}'\boldsymbol{\Omega}^{-1}\mathbf{1} - (\boldsymbol{\beta}'\boldsymbol{\Omega}^{-1}\mathbf{1})^{2}} \tag{9}$$

where $\mathbf{1}$ is a 24×1 vector of ones and $\boldsymbol{\beta}$ is a 24×1 vector of individual stock beta forecasts. This weighting scheme follows from the global minimum variance portfolio, subject to the constraints that the portfolio weights sum to one and the portfolio beta is equal to one. The portfolio is held for one month and its realized return is recorded. This procedure begins when the manager has sufficient data to estimate the covariance matrices and it is repeated at the beginning of every month. The optimal portfolio weights vary through time as the covariance matrix estimate changes. Thus, for each estimation method, the manager has the ex-post performance of its minimum tracking error portfolio, which is rebalanced monthly, and then uses the ex-post beta of its minimum tracking error volatility portfolio as a measure of the precision of the covariance estimator.

We use three different methodologies to estimate the covariance matrix for different beta forecasting models. We start with the covariance matrix from individual stocks' monthly returns. With the monthly returns from the previous 5 years, we construct the portfolio's monthly sample covariance matrix ($\mathbf{\Omega}^R_{t,Monthly}$),

$$\mathbf{\Omega}^R_{Monthly} = \frac{1}{T-1}\sum_{j=1}^{T}(\mathbf{R_{m-j}} - \overline{\mathbf{R}})'(\mathbf{R_{m-j}} - \overline{\mathbf{R}}) \tag{10}$$

where $\mathbf{R_{m-j}}$ is the 1×24 vector of stock monthly returns in the month of $m-j$, ($\overline{\mathbf{R}}$) is the in-sample historical average of these monthly return vectors, and T is the sample size of the estimation window. For example, if we use the monthly returns from the last five years, T is equal to 60. This sample covariance matrix is used to predict the variances and covariances for the next month and also to optimize the weights of each stock in the portfolio as defined in equation (9).

The second monthly covariance is based on daily returns over the previous month as in Liu (2009), where he shows that the monthly covariance matrix can be obtained from the daily returns by simply summing up the daily sample covariance estimates within a month. We denote this estimate as $\mathbf{\Omega}^R_{Daily}$,

$$\mathbf{\Omega}^R_{Daily} = \sum_{j=1}^{N} (\mathbf{R_{j,m}} - \overline{\mathbf{R}}_\mathbf{m})'(\mathbf{R_{j,m}} - \overline{\mathbf{R}}_\mathbf{m}) \tag{11}$$

where $\mathbf{R_{j,m}}$ is the 1×24 vector of stock returns on day j in month m, $\overline{\mathbf{R}}_\mathbf{m}$ is the in-sample daily average returns, and N the number of days in a month.

Thirdly, because of the benefits of using high-frequency data to estimate the covariance matrix, demonstrated in Sheppard (2006) and Liu (2009), we generate the monthly realized covariance matrix ($\mathbf{\Omega}^R_{Intraday}$) using 30 minute and overnight returns, so that we can construct the optimal portfolio from intraday data. We estimate the monthly covariance matrix by using the 30 minute and overnight returns from the previous month:

$$\mathbf{\Omega}^R_{Intraday} = \sum_{j=1}^{T} \sum_{k=1}^{K} \mathbf{R'_{k,j,m}} \times \mathbf{R_{k,j,m}} \tag{12}$$

where $K - 1$ is the total number of 30 minute stock returns in a trading day, T is the total number of trading days in a month, and $\mathbf{R_{k,j,m}}$ is the 1×24 vector of stock returns at interval k on day j in month m. In our sample, we have 13 returns at the 30 minute frequency and one overnight return, per trading day, and about 21 trading days in a month to estimate the monthly covariance matrix.

With the corresponding covariance matrix computed from returns from the same frequency as the returns generating the beta forecasts, we then evaluate the portfolio optimization for the autoregressive beta models and constant beta models. Our evaluation sample is from July 2002 to July 2009 and our criteria of forecasting performance is MSE and MAE from the difference between the monthly portfolio realized beta (computed from 30 minute returns) based on the optimal portfolio weights and the target portfolio beta of one.

The results are shown in table 9, where we report the mean, minimum, maximum, and standard deviation of the realized betas on all the optimal portfolios from different models and data frequencies. We also report the MSE and MAE of the realized betas relative to the target beta of one. In Panel A, we compare different models based on intraday data. The smallest MSE occurs with the high frequency constant beta model using the previous 2 months of 30 minute intraday returns. This approach also has the smallest MAE. The mean of the 2M(30m) constant beta model's realized portfolio beta is 0.995 in the evaluation period, and the MSE is very close to 0. Panel A also shows that the performance from the AR models is worse than the constant beta model based on intraday data. For example, the MSE of 2M(30m) is 0.0009, whereas in the AR(3) case, the MSE is 0.0232.

Panel B of table 9 reports the optimization results based on daily returns. For the constant beta model based on daily data, it actually produces better performance than the autoregressive beta model that uses high frequency 30 minute return data. For example, the mean of the 12M(Daily) model is closer to one, compared to the AR(3) model, and the standard deviation, MSE and MAE of the 12M(Daily) model are much less than those of the AR models. The MSE of the AR(3) model in panel A is 0.0215, and MAE is 0.1210, while the MSE for the 12M(Daily) model is just 0.0014 and MAE is 0.0277. However, the 12M(Daily) model has a higher standard deviation than the 2M(30m) model (over three times higher) which again demonstrates that there are substantial gains from utilizing intraday data for targeted beta portfolio construction.

The panel C of table 9 reports the optimization results based on the monthly data and the Fama-MacBeth model. The results indicate that although the Fama-MacBeth model with monthly data over 60 months cannot produce better performance than the best model based on intraday or daily data in terms of the MSE and MAE of the optimal portfolios, it is better than most of AR models from intraday data. For example, the Fama-MacBeth model with the monthly returns from the last five years has a mean portfolio realized beta closer to one and lower MSE and MAE compared to the AR(3) model. However, the reported MSE from the 60M(Monthly) model is over 18 times higher than the 2M(30m) model.

6 Conclusion

This paper demonstrates that when reliable higher frequency returns are available, these will deliver more accurate one-month-ahead beta forecasts, relative to forecasts from returns measured at a lower frequency. With reliable 30 minute returns, a constant beta model over the prior two months, delivers the most accurate one-month-ahead beta forecast. When the highest reliable return frequency measurement is daily, a constant beta model over the prior twelve months delivers the most accuracy for the one-month-ahead beta forecast. When the highest reliable return frequency measurement is monthly, the Fama-MacBeth constant beta model over the prior 60 months, delivers the most accurate one-month-ahead beta forecast. We also demonstrate that these beta forecasting results extend to portfolio optimization when a desired portfolio beta exposure is being targeted.

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Table 1: **Names of Stocks in the Sample**

NYSE Code	**Company Name**
AA	Alcoa Inc
AXP	American Express Company
MMM	3M Company
BA	The Boeing Company
DD	E.I. du Pont de Nemours & Company
UTX	United Technologies Corporation
CAT	Caterpillar Inc.
BAC	Bank of America Corporation
GE	General Electric Company
CVX	Chevron Corporation
DIS	The Walt Disney Company
HD	The Home Depot, Inc
IBM	International Business Machines Corp
MCD	McDonald's Corporation
MRK	Merck & Co., Inc
HPQ	Hewlett-Packard Company
JNJ	Johnson & Johnson
KO	The Coca-Cola Company
PG	The Procter & Gamble Company
JPM	JPMorgan Chase & Co
PFE	Pfizer Inc
T	AT&T Inc
WMT	Wal-Mart Stores, Inc
XOM	Exxon Mobil Corporation

Table 2: **MSE of One-Month-Ahead Forecast of Dow Betas**

	AR(1)	**AR(3)**	**AR(5)**	**1M(30m)**	**2M(30m)**	**4M(30m)**	**12M(30m)**	**3M(Daily)**	**6M(Daily)**	**12M(Daily)**	**24M(Daily)**	**60M(Monthly)**	**MIDAS12(Weekly)**
AA	0.1034	0.1053	0.1108	0.1081	0.0922	0.1093	0.1287	0.1438	0.1078	**0.0875**	0.0894	0.1861	0.1183
AXP	0.0986	0.0789	0.0838	**0.0584**	0.0803	0.1419	0.1252	0.2034	0.1487	0.1084	0.1116	0.2106	0.1204
MMM	0.0202	0.0221	0.0199	0.0310	0.0281	**0.0196**	0.0200	0.0458	0.0389	0.0284	0.0258	0.0269	0.0257
BA	0.0338	0.0261	0.0294	0.0315	**0.0261**	0.0270	0.0377	0.0645	0.0508	0.0462	0.0397	0.0453	0.0343
DD	0.0311	0.0304	0.0339	0.0363	**0.0282**	0.0520	0.0773	0.0404	0.0429	0.0602	0.0588	0.0348	0.0380
UTX	0.0277	0.0291	0.0317	**0.0219**	0.0270	0.0257	0.0281	0.0374	0.0310	0.0280	0.0279	0.0351	0.0298
CAT	0.0668	0.0586	0.0608	0.0515	**0.0465**	0.0648	0.1020	0.0710	0.0717	0.0956	0.1099	0.0711	0.0715
BAC	0.3519	0.2659	0.3319	0.2573	0.3154	0.4998	0.3669	0.7618	0.5994	0.3630	0.3952	0.9021	**0.2157**
GE	0.0807	0.0745	0.0797	0.0651	**0.0629**	0.0762	0.0875	0.0675	0.0928	0.0898	0.0910	0.0875	0.0790
CVX	0.1046	0.0921	0.1031	0.0889	**0.0836**	0.1187	0.1192	0.1131	0.1344	0.1209	0.1090	0.2129	0.1244
DIS	0.0327	0.0322	0.0295	0.0442	0.0309	**0.0233**	0.0265	0.0654	0.0408	0.0286	0.0243	0.0806	0.0420
HD	0.0825	0.0849	0.0849	0.0912	0.0791	**0.0706**	0.0722	0.1061	0.0926	0.0781	0.0833	0.1481	0.0989
IBM	0.0177	0.0168	**0.0161**	0.0214	0.0202	0.0189	0.0176	0.0661	0.0419	0.0333	0.0226	0.2216	0.0703
MCD	0.0283	0.0298	0.0289	0.0326	**0.0247**	0.0276	0.0282	0.0551	0.0559	0.0344	0.0378	0.1314	0.0559
MRK	0.0301	**0.0299**	0.0310	0.0395	0.0367	0.0358	0.0313	0.0696	0.0561	0.0465	0.0399	0.0550	0.0404
HPQ	0.0225	0.0320	0.0280	0.0429	0.0330	0.0210	**0.0208**	0.0675	0.0289	0.0240	0.0216	0.3257	0.1031
JNJ	0.0270	0.0276	0.0281	0.0221	**0.0220**	0.0241	0.0277	0.0236	0.0270	0.0290	0.0232	0.0452	0.0248
KO	0.0140	0.0132	0.0140	0.0144	**0.0118**	0.0139	0.0180	0.0189	0.0182	0.0206	0.0233	0.0263	0.0186
PG	0.0117	0.0113	0.0123	0.0113	0.0093	**0.0089**	0.0097	0.0304	0.0168	0.0118	0.0154	0.1167	0.0382
JPM	0.1493	**0.1048**	0.1317	0.1482	0.1519	0.2282	0.1572	0.4154	0.3310	0.2016	0.2165	0.3378	0.2028
PFE	0.0280	0.0309	0.0274	0.0367	0.0311	0.0271	**0.0233**	0.0657	0.0492	0.0445	0.0479	0.0624	0.0431
T	0.0372	0.0398	0.0399	0.0458	**0.0351**	0.0387	0.0477	0.0489	0.0431	0.0491	0.0552	0.3113	0.1104
WMT	0.0724	0.0623	0.0661	0.0577	0.0630	0.0613	0.0579	0.0701	0.0592	**0.0547**	0.0643	0.2682	0.1144
XOM	0.0772	**0.0618**	0.0656	0.0751	0.0672	0.0803	0.0851	0.1038	0.0974	0.0937	0.0845	0.1932	0.1017

The $AR(p)$ forecast is based on the previous 48 months of realized beta (computed from 30 minute returns over the month.) The 1M(30m) forecast is the realized beta computed from 30 minute returns over the previous month. Similarly, the 2M(30m) forecast is the realized beta computed from 30 minute returns over the previous 2 months, and so on. The 3M(Daily) forecast is the realized beta computed from daily returns over the previous 3 months. Similarly, the 6M(Daily) forecast is the realized beta computed from daily returns over the previous 6 months, and so on. The 60M(Monthly) is the Fama-MacBeth forecast based on the previous 5 years of monthly returns. The MIDAS12(Weekly) is the MIDAS forecast with 12 lags of weekly realized beta. The minimum MSE for each stock is in bold. The forecast evaluation covers the period May 2006 through to July 2009.

Table 3: **MAE of One-Month-Ahead Forecast of Dow Betas**

	AR(1)	AR(3)	AR(5)	1M(30m)	2M(30m)	4M(30m)	12M(30m)	3M(Daily)	6M(Daily)	12M(Daily)	24M(Daily)	60M(Monthly)	MIDAS12(Weekly)
AA	0.2467	0.2511	0.2550	0.2505	0.2413	0.2614	0.2862	0.2923	0.2408	**0.2300**	0.2360	0.3609	0.2408
AXP	0.2286	0.2117	0.2195	**0.1885**	0.2201	0.2775	0.2665	0.2882	0.2685	0.2476	0.2562	0.3976	0.2265
MMM	0.1166	0.1212	0.1173	0.1409	0.1356	**0.1167**	0.1167	0.1702	0.1566	0.1456	0.1373	0.1463	0.1341
BA	0.1454	0.1288	0.1396	0.1413	**0.1253**	0.1293	0.1595	0.1897	0.1655	0.1754	0.1694	0.1761	0.1431
DD	0.1453	0.1382	0.1482	0.1439	**0.1331**	0.1787	0.1998	0.1658	0.1791	0.1898	0.1793	0.1418	0.1537
UTX	0.1375	0.1425	0.1447	**0.1198**	0.1358	0.1258	0.1295	0.1563	0.1392	0.1289	0.1278	0.1422	0.1357
CAT	0.2011	0.1929	0.1929	0.1861	**0.1744**	0.2034	0.2556	0.2161	0.2149	0.2433	0.2679	0.1983	0.2036
BAC	0.4188	0.3559	0.3875	0.3578	0.3787	0.4645	0.3951	0.5911	0.5006	0.4296	0.4575	0.7775	**0.3157**
GE	0.2040	0.2114	0.2072	0.2015	**0.1850**	0.1937	0.2085	0.2112	0.2189	0.2150	0.2111	0.2275	0.2038
CVX	0.2272	0.2293	0.2393	0.2285	**0.2118**	0.2673	0.2636	0.2413	0.2755	0.2756	0.2576	0.3902	0.2389
DIS	0.1197	0.1248	0.1196	0.1616	0.1327	**0.1040**	0.1177	0.1895	0.1623	0.1263	0.1109	0.2363	0.1279
HD	0.2192	0.2303	0.2226	0.2188	0.2019	**0.1868**	0.1989	0.2541	0.2235	0.2165	0.2202	0.3270	0.2163
IBM	0.1106	0.1085	0.1073	0.1123	0.1174	**0.1054**	0.1061	0.2235	0.1606	0.1449	0.1171	0.4138	0.1236
MCD	0.1350	0.1408	0.1373	0.1490	**0.1263**	0.1316	0.1328	0.1866	0.1862	0.1513	0.1561	0.3029	0.1394
MRK	0.1478	0.1445	0.1470	0.1615	0.1583	0.1514	**0.1411**	0.2024	0.1944	0.1718	0.1601	0.1916	0.1587
HPQ	0.1275	0.1444	0.1359	0.1629	0.1368	0.1142	**0.1123**	0.1922	0.1365	0.1149	0.1191	0.5161	0.1320
JNJ	0.1266	0.1228	0.1262	0.1173	**0.1085**	0.1184	0.1337	0.1140	0.1268	0.1384	0.1225	0.1641	0.1232
KO	0.0939	0.0940	0.0969	0.0895	**0.0831**	0.0921	0.1106	0.1062	0.1027	0.1133	0.1202	0.1382	0.0968
PG	0.0935	0.0851	0.0919	0.0794	0.0759	**0.0702**	0.0817	0.1215	0.0914	0.0887	0.1099	0.3074	0.0832
JPM	0.3191	**0.2515**	0.2864	0.2949	0.2928	0.3534	0.2992	0.4669	0.4213	0.3569	0.3743	0.4737	0.3004
PFE	0.1338	0.1434	0.1367	0.1517	0.1447	0.1314	**0.1245**	0.1957	0.1764	0.1627	0.1734	0.2051	0.1503
T	0.1514	0.1497	0.1506	0.1596	**0.1471**	0.1585	0.1735	0.1737	0.1639	0.1706	0.1712	0.5045	0.1558
WMT	0.1887	0.1980	0.1942	0.1708	0.1701	0.1799	0.1679	0.1977	0.1699	**0.1516**	0.1627	0.4551	0.1732
XOM	0.2062	0.1988	0.2004	0.2069	**0.1978**	0.2369	0.2407	0.2437	0.2547	0.2587	0.2434	0.3806	0.2184

The $AR(p)$ forecast is based on the previous 48 months of realized beta (computed from 30 minute returns over the month.) The 1M(30m) forecast is the realized beta computed from 30 minute returns over the previous month. Similarly, the 2M(30m) forecast is the realized beta computed from 30 minute returns over the previous 2 months, and so on. The 3M(Daily) forecast is the realized beta computed from daily returns over the previous 3 months. Similarly, the 6M(Daily) forecast is the realized beta computed from daily returns over the previous 6 months, and so on. The 60M(Monthly) is the Fama-MacBeth forecast based on the previous 5 years of monthly returns. The MIDAS12(Weekly) is the MIDAS forecast with 12 lags of weekly realized beta. The minimum MAE for each stock is in bold. The forecast evaluation covers the period May 2006 through to July 2009.

Table 4: **Dow Stocks MSE for One-Month-Ahead Beta Forecasts**

	n	**AR(1)**	**AR(2)**	**AR(3)**	**AR(4)**	**AR(5)**
A	**24**	0.0682	0.0686	0.0598	0.0624	0.0656
	48	0.0646	0.0650	**0.0567**	0.0590	0.0620
	72	0.0694	0.0691	0.0608	0.0630	0.0662
	100	0.0708	0.0696	0.0628	0.0647	0.0673
	1M(30m)	**2M(30m)**	**4M(30m)**	**6M(30m)**	**12M(30m)**	**18M(30m)**
B	0.0597	**0.0586**	0.0705	0.0756	0.0715	0.0741
	1M(Daily)	**3M(Daily)**	**6M(Daily)**	**12M(Daily)**	**24M(Daily)**	**48M(Daily)**
C	0.1774	0.1148	0.0948	**0.0741**	0.0758	0.0901
	24M(Monthly)	**36M(Monthly)**	**48M(Monthly)**	**60M(Monthly)**	**72M(Monthly)**	**80M(Monthly)**
D	0.1924	0.1782	0.1693	**0.1563**	0.1678	0.1758
	MIDAS2(Weekly)	**MIDAS4(Weekly)**	**MIDAS8(Weekly)**	**MIDAS12(Weekly)**	**MIDAS16(Weekly)**	**MIDAS20(Weekly)**
E	0.1532	0.1372	0.1115	**0.1020**	0.1150	0.1233

The $AR(p)$ forecast is based on the previous n months of realized beta (computed from 30 minute returns over the month.) The 1M(30m) forecast is the realized beta computed from 30 minute returns over the previous month. Similarly, the 2M(30m) forecast is the realized beta computed from 30 minute returns over the previous 2 months, and so on. The 1M(Daily) forecast is the realized beta computed from daily returns over the previous month. Similarly, the 3M(Daily) forecast is the realized beta computed from daily returns over the previous 3 months, and so on. The 24M(Monthly) is the Fama-MacBeth forecast based on the previous 24 monthly returns. Similarly, the 36M(Monthly) is the Fama-MacBeth forecast based on the previous 36 monthly returns, and so on. The MIDAS2(Weekly) is the MIDAS forecast with 2 lags of weekly realized beta. Similarly, The MIDAS4(Weekly) is the MIDAS forecast with 4 lags of weekly realized beta, and so on. Average values are computed by taking the mean over the 24 stocks and the minimum values for each category are in bold. The forecast evaluation covers the period May 2006 through to July 2009.

Table 5: **Dow Stocks MAE for One-Month-Ahead Beta Forecasts**

	n	**AR(1)**	**AR(2)**	**AR(3)**	**AR(4)**	**AR(5)**
A	**24**	0.1819	0.1796	0.1737	0.1763	0.1765
	48	0.1779	0.1768	**0.1717**	0.1730	0.1752
	72	0.1845	0.1811	0.1753	0.1774	0.1783
	100	0.1865	0.1824	0.1763	0.1783	0.1792
	1M(30m)	**2M(30m)**	**4M(30m)**	**6M(30m)**	**12M(30m)**	**18M(30m)**
B	0.1748	**0.1681**	0.1762	0.1813	0.1842	0.1867
	1M(Daily)	**3M(Daily)**	**6M(Daily)**	**12M(Daily)**	**24M(Daily)**	**48M(Daily)**
C	0.2873	0.2246	0.2054	**0.1923**	0.1942	0.2116
	24M(Monthly)	**36M(Monthly)**	**48M(Monthly)**	**60M(Monthly)**	**72M(Monthly)**	**80M(Monthly)**
D	0.3091	0.2796	0.2678	**0.2394**	0.2587	0.2814
	MIDAS2(Weekly)	**MIDAS4(Weekly)**	**MIDAS8(Weekly)**	**MIDAS12(Weekly)**	**MIDAS16(Weekly)**	**MIDAS20(Weekly)**
E	0.2671	0.2441	0.2295	**0.2136**	0.2267	0.2396

The $AR(p)$ forecast is based on the previous n months of realized beta (computed from 30 minute returns over the month.) The 1M(30m) forecast is the realized beta computed from 30 minute returns over the previous month. Similarly, the 2M(30m) forecast is the realized beta computed from 30 minute returns over the previous 2 months, and so on. The 1M(Daily) forecast is the realized beta computed from daily returns over the previous month. Similarly, the 3M(Daily) forecast is the realized beta computed from daily returns over the previous 3 months, and so on. The 24M(Monthly) is the Fama-MacBeth forecast based on the previous 24 monthly returns. Similarly, the 36M(Monthly) is the Fama-MacBeth forecast based on the previous 36 monthly returns, and so on. The MIDAS2(Weekly) is the MIDAS forecast with 2 lags of weekly realized beta. Similarly, The MIDAS4(Weekly) is the MIDAS forecast with 4 lags of weekly realized beta, and so on. Average values are computed by taking the mean over the 24 stocks and the minimum values for each category are in bold. The forecast evaluation covers the period May 2006 through to July 2009.

Table 6: **Diebold-Mariano Test on Squared Errors**

Company	**2M(30m) vs 12M(Daily)**	**2M(30m) vs 60M(Monthly)**	**12M(Daily) vs 60M(Monthly)**	**2M(30m) vs AR(3)**	**AR(3) vs 12M(Daily)**	**AR(3) vs 60M(Monthly)**
AA	0.2374	**0.0129**	**0.0134**	0.7837	0.2701	**0.0188**
AXP	0.0792	**0.0065**	**0.0228**	**0.0000**	0.1185	**0.0235**
MMM	**0.0070**	**0.0000**	**0.0000**	0.2721	**0.0001**	**0.0000**
BA	0.3037	**0.0002**	**0.0060**	0.1830	**0.0002**	**0.0000**
DD	0.3588	**0.0237**	**0.0017**	**0.0020**	0.2299	0.9252
UTX	**0.0919**	**0.0000**	0.3897	**0.0000**	0.0629	**0.0000**
CAT	0.1469	0.1910	**0.0203**	**0.0153**	0.0504	0.5969
BAC	0.7805	0.4805	0.3550	0.4752	0.8358	0.4652
GE	0.0828	**0.0223**	0.3021	0.4290	**0.0020**	**0.0149**
CVX	0.5752	0.1540	**0.0943**	**0.0455**	0.4895	**0.0177**
DIS	0.2091	**0.0021**	**0.0318**	0.8474	**0.0011**	**0.0013**
HD	0.4722	0.1425	0.0189	0.8899	**0.0296**	0.1832
IBM	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0001**	**0.0000**
MCD	**0.0000**	**0.0075**	0.2591	0.0913	**0.0002**	**0.0033**
MRK	0.2106	**0.0000**	**0.0005**	**0.0499**	0.1970	**0.0000**
HPQ	0.0706	**0.0287**	0.1724	0.0834	**0.0117**	0.1141
JNJ	0.3761	**0.0000**	**0.0000**	**0.0239**	**0.0000**	**0.0000**
KO	**0.0352**	**0.0000**	**0.0000**	**0.0000**	0.0599	**0.0000**
PG	**0.0004**	**0.0000**	**0.0000**	**0.0011**	**0.0342**	**0.0000**
JPM	0.2011	0.3169	0.2414	0.5401	0.0796	0.2701
PFE	0.3060	**0.0015**	**0.0026**	0.8488	**0.0000**	**0.0002**
T	0.3416	**0.0066**	0.1497	0.0728	**0.0000**	**0.0052**
WMT	0.4377	**0.0012**	**0.0000**	**0.0390**	0.1888	**0.0003**
XOM	0.1927	**0.0000**	**0.0092**	0.4803	**0.0123**	**0.0000**

This table presents the p-value of the Diebold-Mariano test statistics on squared forecasting errors for comparing forecasting accuracy for 6 paris of beta estimation models of *one*-month-ahead forecasting. Values in bold represent *two*-tailed rejection of equal predictive accuracy at the 5% confidence level. The AR(3) forecast is based on the previous 48 months of realized beta (computed from 30 minute returns over the month.) The 2M(30m) forecast is the realized beta computed from 30 minute returns over the previous 2 months. The 12M(Daily) forecast is the realized beta computed from daily returns over the previous 12 months. The 60M(Monthly) is the Fama-MacBeth forecast based on the previous 5 years of monthly returns. The forecast evaluation covers the period July 2002 through to July 2009.

Table 7: **Diebold-Mariano Test on Absolute Errors**

Company	**2M(30m) vs 12M(Daily)**	**2M(30m) vs 60M(Monthly)**	**12M(Daily) vs 60M(Monthly)**	**2M(30m) vs AR(3)**	**AR(3) vs 12M(Daily)**	**AR(3) vs 60M(Monthly)**
AA	0.2785	**0.0077**	**0.0136**	0.8418	0.3664	**0.0309**
AXP	0.0649	**0.0087**	**0.0210**	**0.0107**	0.2244	**0.0169**
MMM	**0.0423**	**0.0047**	**0.0180**	**0.0466**	0.0997	**0.0007**
BA	0.0895	0.0554	0.4588	0.5477	**0.0255**	**0.0163**
DD	0.1533	0.3806	0.2944	0.3116	**0.0475**	0.6841
UTX	0.2981	**0.0149**	**0.0028**	**0.0036**	0.1338	**0.0038**
CAT	0.2311	0.5730	0.6383	0.6349	0.3537	0.6914
BAC	0.7048	0.1738	**0.0404**	0.3864	0.9457	0.1514
GE	0.1945	**0.0218**	**0.0466**	**0.0463**	0.2072	0.0769
CVX	0.1380	0.1795	0.5382	**0.0003**	0.0704	0.0864
DIS	**0.0318**	**0.0028**	0.4550	0.5739	**0.0082**	**0.0003**
HD	0.4375	0.1410	0.1062	0.8191	0.1815	**0.0284**
IBM	0.3145	**0.0000**	**0.0005**	**0.0268**	**0.0090**	**0.0000**
MCD	**0.0146**	**0.0430**	**0.0107**	**0.0322**	**0.0001**	**0.0097**
MRK	0.2127	**0.0066**	**0.0195**	0.6003	0.1561	**0.0003**
HPQ	0.3641	**0.0174**	0.1335	0.6245	0.1865	**0.0062**
JNJ	**0.0044**	**0.0000**	**0.0032**	**0.0003**	**0.0173**	**0.0001**
KO	**0.0003**	**0.0000**	**0.0020**	**0.0085**	**0.0000**	**0.0000**
PG	**0.0008**	**0.0000**	**0.0000**	0.2315	**0.0004**	**0.0000**
JPM	0.2575	**0.0505**	0.0766	0.4436	0.1725	**0.0460**
PFE	0.1411	**0.0004**	**0.0080**	0.9274	**0.0160**	**0.0001**
T	0.0564	0.0547	**0.0051**	0.8318	**0.0093**	**0.0434**
WMT	0.3676	**0.0000**	**0.0000**	**0.0500**	0.7990	**0.0000**
XOM	**0.0332**	**0.0003**	**0.0187**	0.4698	**0.0081**	**0.0000**

This table presents the p-value of the Diebold-Mariano test statistics on absolute value of forecasting errors for comparing forecasting accuracy for 6 paris of beta estimation models of *one*-month-ahead forecasting. Values in bold represent *two*-tailed rejection of equal predictive accuracy at 5% confidence level. The AR(3) forecast is based on the previous 48 months of realized beta (computed from 30 minute returns over the month.) The 2M(30m) forecast is the realized beta computed from 30 minute returns over the previous 2 months. The 12M(Daily) forecast is the realized beta computed from daily returns over the previous 12 months. The 60M(Monthly) is the Fama-MacBeth forecast based on the previous 5 years of monthly returns. The forecast evaluation covers the period July 2002 through to July 2009.

Table 8: **Dow Stocks' Risk Ranking by Alternate Beta Forecasts for July 2009**

Company	**Forecast**				**Rank**			
	60M(Monthly)	**AR(3)**	**12M(Daily)**	**2M(30m)**	**2M(30m)**	**12M(Daily)**	**AR(3)**	**60M(Monthly)**
MCD	0.7061	0.6411	0.6712	0.5031	1	4	4	7
WMT	0.2653	0.5761	0.5982	0.5081	2	1	3	1
JNJ	0.6123	0.5391	0.6032	0.5342	3	2	1	3
KO	0.6440	0.5621	0.6352	0.6353	4	3	2	4
PG	0.6674	0.7893	0.6942	0.7671	5	5	6	5
T	0.7154	0.9062	0.9613	0.8341	6	11	8	8
PFE	0.7972	0.7264	0.8362	0.8432	7	7	5	10
IBM	0.8724	0.9261	0.7921	0.9051	8	6	10	12
MRK	0.8644	0.8850	0.8991	0.9473	9	9	7	11
XOM	0.5151	0.9134	1.0962	0.9663	10	15	9	2
MMM	0.9001	0.9982	0.8553	0.9794	11	8	13	13
HPQ	1.0974	0.9773	0.9352	1.0131	12	10	12	15
CVX	0.7044	0.942	1.230	1.0333	13	18	11	6
UTX	1.0301	1.1562	1.0434	1.1073	14	12	14	14
BA	1.347	1.3172	1.0763	1.2260	15	14	17	18
HD	0.7301	1.2682	1.0450	1.2530	16	13	16	9
DIS	1.1231	1.2222	1.2380	1.2574	17	19	15	16
JPM	1.2893	1.5964	1.9131	1.3432	18	23	19	17
BAC	2.9081	2.1852	2.5403	1.6014	19	24	24	24
AXP	2.0721	1.4903	1.8022	1.6334	20	21	18	22
GE	1.7051	1.7382	1.2513	1.7074	21	20	20	20
DD	1.5070	1.8631	1.2142	1.7784	22	17	22	19
CAT	2.0301	1.8092	1.2102	1.8081	23	16	21	21
AA	2.2934	1.9761	1.8543	1.9511	24	22	23	23

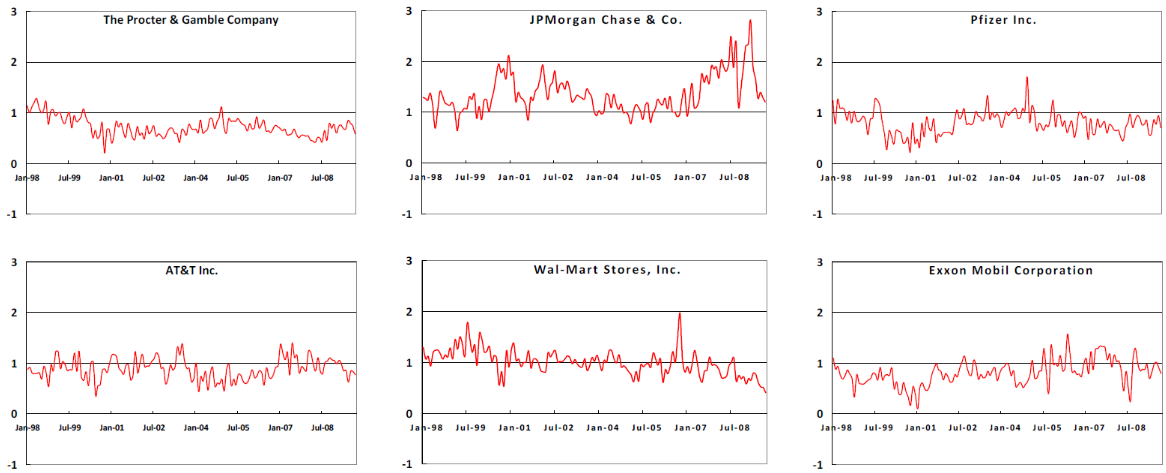
The 60M(Monthly) is the Fama-MacBeth forecast based on the previous 5 years of monthly returns. The AR(3) forecast is based on the previous 48 months of realized beta (computed from 30 minute returns over the month). The 12M(Daily) forecast is the realized beta computed from daily returns over the previous 12 months. And the 2M(30m) forecast is the realized beta computed from 30 minute returns over the previous 2 months.

Table 9: **Realized Betas of Optimal Portfolios Targeting Beta of One**

	Model	Mean	Min	Max	Stdev	MSE	MAE
A	**1M(30m)**	0.9643	0.7337	1.1326	0.0630	0.0061	0.0562
	2M(30m)	**0.9988**	0.9412	1.0375	**0.0226**	**0.0009**	**0.0193**
	3M(30m)	0.9485	0.5858	1.1444	0.0802	0.0105	0.0759
	4M(30m)	0.9384	0.6312	1.1315	0.0874	0.0138	0.0879
	6M(30m)	0.9164	0.5761	1.1166	0.0892	0.0177	0.1053
	8M(30m)	0.9037	0.6166	1.1031	0.0834	0.0184	0.1106
	10M(30m)	0.8976	0.6198	1.0681	0.0786	0.0193	0.1161
	12M(30m)	0.8879	0.6250	1.0560	0.0801	0.0211	0.1219
	14M(30m)	0.8813	0.5985	1.0507	0.0825	0.0235	0.1299
	16M(30m)	0.8749	0.5982	1.0447	0.0830	0.0247	0.1344
	18M(30m)	0.8697	0.5964	1.0418	0.0854	0.0267	0.1401
	20M(30m)	0.8666	0.5885	1.0382	0.0868	0.0287	0.1464
	22M(30m)	0.8605	0.5858	1.0370	0.0877	0.0307	0.1528
	24M(30m)	0.8573	0.5833	1.0336	0.0899	0.0327	0.1580
	AR(1)	0.8812	0.6362	1.0244	0.0912	0.0269	0.1370
	AR(2)	0.8931	0.6209	1.0225	0.0889	0.0232	0.1248
	AR(3)	0.9028	0.6201	1.0405	0.0848	0.0215	0.1210
	AR(4)	0.8986	0.6339	1.0178	0.0863	0.0220	0.1213
	AR(5)	0.8996	0.6552	1.0175	0.0817	0.0204	0.1176
B	**1M(Daily)**	0.7751	0.7092	1.0106	0.0736	0.0603	0.2248
	2M(Daily)	0.8374	0.5784	1.0368	0.0827	0.0333	0.1638
	3M(Daily)	0.8751	0.4435	0.9571	0.0995	0.0209	0.1251
	6M(Daily)	0.9413	0.6189	1.1247	0.0996	0.0083	0.0731
	12M(Daily)	**0.9901**	0.7410	1.1173	**0.0718**	**0.0014**	**0.0277**
	18M(Daily)	0.9602	0.8102	1.0988	0.0552	0.0046	0.0538
	24M(Daily)	0.9271	0.9273	1.1346	0.0358	0.0104	0.0849
	48M(Daily)	0.8501	0.7717	1.1774	0.0702	0.0323	0.1566
C	**24M(Monthly)**	0.9081	0.5392	1.5261	0.1573	0.0382	0.1502
	36M(Monthly)	0.9472	0.5633	1.6131	0.1631	0.0431	0.1387
	48M(Monthly)	0.9678	0.6463	1.6595	0.1362	0.0242	0.0968
	60M(Monthly)	**0.9964**	0.8864	1.7011	**0.0981**	**0.0167**	**0.0689**
	72M(Monthly)	0.9761	0.5572	1.4991	0.1441	0.0249	0.1185
	80M(Monthly)	0.9645	0.6036	1.6983	0.1572	0.0368	0.1193

The 1M(30m) forecast is the realized beta computed from 30 minute returns over the previous month. Similarly, the 2M(30m) forecast is the realized beta computed from 30 minute returns over the previous 2 months, and so on. The $AR(p)$ forecast is based on the previous 48 months of realized beta (computed from 30 minute returns over the month.) The 1M(Daily) forecast is the realized beta computed from daily returns over the previous month. Similarly, the 2M(Daily) forecast is the realized beta computed from daily returns over the previous 2 months, and so on. The 24M(Monthly) is the Fama-MacBeth forecast based on the previous 24 monthly returns. Similarly, the 36M(Monthly) is the Fama-MacBeth forecast based on the previous 36 monthly returns, and so on. The optimal result for each return measurement setting is in bold. The portfolio optimization evaluation covers the period July 2002 through to July 2009.

Figure 1: **Monthly Realized Betas for Dow Stocks**



Note: The realized beta is computed from 30 minute returns over the month. The sample covers the period from January 1998 to July 2009.