Performance measures of models to predict Loss Given Default: a critical review

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Outline

• Introduction

• Loss Given Default (LGD)

• Models to predict LGD

• Performance measures
  – Error measures
  – Other measures

• Conclusions
Loss Given Default (LGD)

• The lender’s loss on a loan due to the customer’s default, i.e. failure to meet the credit commitment

• “The ratio of the loss on an exposure due to the default of a counterparty to the amount outstanding at default” (Article 4(27) of the Council Directive 2006/48/EC)

• Basel II and III
  – Under the Advanced Internal Ratings-Based (AIRB) approach, lenders are allowed to use their own predictions of risk parameters, including LGD
LGD distribution example
Models to predict LGD

- Unsecured loans
  - One-stage models
  - Multi-stage approaches
    - Separation of 0s (+ Separation of 1s) + Prediction
- Mortgage loans
  - One-stage models
  - Two-stage approaches
    - Repossession model + Haircut model
Models to predict LGD

- Separation stage(s)
  - Logistic regression
  - Decision trees

- Prediction stage/One-stage models
  - Regression models
  - Tobit models
  - Survival analysis
  - Classification and Regression Trees (CART)
  - Other nonlinear models
Performance measures

- Credit scoring
  - Gini coefficient
  - Kolmogorov-Smirnov (KS) statistic

- LGD
  - ???
Error measures: MSE

- Mean Square Error (MSE):

\[ MSE = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \]

- Sensitive to extreme values of the residuals
- E.g. Bellotti and Crook (2008)
Error measures: RMSE

• Root Mean Square Error (RMSE):

\[ \text{RMSE} = \sqrt{\text{MSE}} \]

• Expressed in the same units as LGD

• Bastos (2010)
Error measures: MAE

- Mean Absolute Error (MAE) a.k.a. Mean Absolute Deviation (MAD):

\[
MAE = \frac{1}{n} \sum_{i=1}^{n} |y_i - \hat{y}_i|
\]

- Expressed in the same units as LGD
- Compare with RMSE
- E.g. Bellotti and Crook (2008)
Error measures: RAE

- Relative Absolute Error (RAE):

\[ RAE = \frac{\sum_{i=1}^{n} |y_i - \hat{y}_i|}{\sum_{i=1}^{n} |y_i - \bar{y}_i|} \]

- Ratio of MAE of the model and MAE of a simple predictor

- E.g. Bastos (2010)
Error measures: AOC

- Regression Error Characteristic (REC) curve estimates the CDF of the squared or absolute residual.

- Area Over the REC Curve (AOC) estimates the expected regression error (Bi and Bennett, 2003).

- If the REC curve is derived using the squared (absolute) residuals, then AOC $\rightarrow$ MSE (MAE) as the sample size $\rightarrow \infty$. 
Error measures: AOC

• Loterman et al. (2012) calculated both RMSE and AOC (based on the squared residuals)
  – LGD models: 24 various techniques and six datasets
  – Differences between AOC and the squared RMSE:
    • < 0.001 for five larger datasets
    • < 0.01 for the smallest dataset (test: ca. 1100 loans)

• We recommend applying either AOC or MSE/MAE in order to avoid information redundancy
Other measures: R-squared

- Coefficient of determination (R-squared):

\[ R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y}_i)^2} \]

- E.g. Loterman et al. (2012)

- In an OLS regression model with a constant term, R-squared can be interpreted as the proportion of variation in LGD that is explained by variation in the regressors

- We only recommend using R-squared in OLS models
Other measures: Adjusted R-squared

• Adjusted coefficient of determination (adjusted $R^2$):

$$\bar{R}^2 = 1 - (1 - R^2) \frac{n - 1}{n - k - 1}$$

• Corrected for the number of regressors ($k$)

• Useful when comparing a number of linear LGD models

• E.g. Caselli et al. (2008)
Other measures: Correlation coefficients

- Measure correlation between the observed and predicted LGD (Loterman et al., 2012)

- Pearson’s correlation coefficient:

\[ r = \frac{\sum_{i=1}^{n} (y_i - \bar{y})(\hat{y}_i - \hat{y})}{\sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2 \sum_{i=1}^{n} (\hat{y}_i - \hat{y})^2}} \]

- Measures the strength of the linear relationship between the observed and predicted LGD \((r^2 = R^2 \text{ in OLS models})\)

- Spearman’s and Kendall’s correlation coefficients
Other measures: AUC

- Loans need to be classified into two groups based on the observed LGD, e.g. below-the-mean and over-the-mean
- CDFs of the predicted LGD are computed for the groups
- Receiver Operating Characteristic (ROC) curve is drawn by plotting the CDFs against each other
- Area Under the ROC Curve (AUC) measures how well the model separates loans belonging to the two groups
- E.g. Gupton and Stein (2005)
Proposed measure: MAUC

- AUC has a drawback when applied to LGD as it requires an arbitrary classification of the dependent variable
- \( m \) – the number of unique values of the observed LGD
- Mean AUC (MAUC) is calculated as the average of AUC for all possible divisions into two groups:
  \[
  MAUC = \frac{1}{m-1} \sum_{j=1}^{m-1} AUC_j
  \]
- MAUC takes values from the interval \([0.5, 1]\) like AUC
Example

- Two-stage model applied to the data on personal loans granted by a large UK bank

<table>
<thead>
<tr>
<th>Measure</th>
<th>Value</th>
<th>Measure</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSE</td>
<td>0.143</td>
<td>Spearman</td>
<td>0.255</td>
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<tr>
<td>MAE</td>
<td>0.329</td>
<td>Kendall</td>
<td>0.179</td>
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<td>R-squared</td>
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<td>AUC</td>
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<tr>
<td>Pearson</td>
<td>0.268</td>
<td>MAUC</td>
<td>0.616</td>
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</table>
Example

AUC for different divisions into two groups
Conclusions

• Recommendations for LGD model developers/users
  – Apply either AOC or MSE/MAE
  – Only use R-squared in OLS models
  – Look for an alternative to AUC

• Further research
  – MAUC computed as the weighted average of AUC
  – Impact of segmentation on performance of LGD models
References


Thank you!