Autoregressive Integrated Moving Average with Explanatory Variable (ARIMAX) Model for Thailand Export

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Abstract

This paper considers a univariate time series model to forecast Thailand exports to major trade partners. The main question is whether the explanatory variable, i.e. trade partners' leading indicator, help improving forecasting performance. Specifically, we compare integrated autoregressive moving average (*ARIMA*) and *ARIMA* with explanatory variable. We find that for exports to China, European Union (27 countries) and the United State, the *ARIMA* model with leading indicator outperforms the *ARIMA* model. Moreover, at the Thailand Ministry of Commerce, the practitioners try to forecast the disaggregated export by commodities for each export destination then they combined those series for the country's export forecast. We called this approach "indirect" forecast but we suspect that whether this procedure provides more accurate forecast than modeling the aggregate countrys export, called "direct" forecast. From our rolling out-of-sample forecasts, we find that the indirect forecasts do not outperform the direct forecasts in all trade partners considered.

Keywords: Evaluating forecasts; *ARIMAX* model; Export; Leading indicator

1 Introduction

Between 1993 and 2011, export is accounted for about a half of Thailand GDP. Growth and economic volatility are heavily driven by exports. As a result, the Ministry of Commerce, as a main government agency to promote export and lessen negative shock on the export sector, requires appropriate forecasting models to forecast changes in Thailand export.

One of the main forecasting models used by the Ministry is linear time series model using Box-Jenkins approach or Integrated Autoregressive Moving Average (*ARIMA*) model. However, *ARIMA* model does not capture some turning points in export data. In order to improve forecasting performance, should we include other explanatory variable in to *ARIMA* model, i.e. *ARIMAX* model? The first question, we compare

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the forecasting performance of *ARIMA* and *ARIMAX* model for Thailand export data by trade partners and main export commodities.

For particular trade partners, sometimes, practitioners at the Ministry implement *ARIMA* model for each main commodities and then combine them to be that country's export forecast, called *indirect* forecast. Does this method provide the better forecasting performance than implementing *ARIMA* model to country's export, called *direct* forecast? Hubrich (2005)[1] found that it depends on the Data Generating Process. Specifically, the second objective is to investigate the accuracy of direct and indirect forecasting approaches using Thailand export data.

2 Framework and Methodology

2.1 Time Series Model

Time series models use the past movements of variables in order to predict their future values. Unlike structural models that relate the variable we want to forecast with a set of other variables, the time series model is not based on economic theory. However, in term of forecasting, the reliability of the estimated equation should be based on out-of-sample performance (Stock and Watson, 2003). The time series model can mostly produce quite accurate forecasts, especially in case that there are multidimensional relationships among variables. Because of the complexity of international economic relations, large structural models are likely to suffer from omitted variable bias, misspecifications, simultaneous causality and other problems leading to substantial forecasting errors (Keck, Raubold and Truppia, 2009).

The time series model using Box-Jenkins approach has been proposed by Box and Jenkins (1970). This approach has been widely used in the literature because of its performance and simplicity. Most time series can be described by Autoregressive Moving Average (*ARMA*) model. The stationary series Y_t is said to be $ARMA(p,q)$ if

$$Y_t - \phi_1 Y_{t-1} - ... - \phi_p Y_{t-p} = \varepsilon_t + \theta_1 \varepsilon_{t-1} + ... + \theta_q \varepsilon_{t-q} \tag{1}$$

where ε_t is white noise and there is no common factor between autoregressive polynomial, $(1 - \phi_1 L - \phi_2 L^2 - ... - \phi_p L^p)$, and moving average polynomial, $(1 + \theta_1 L + ... + \theta_q L^q)$, where L is a lag operator. Also, these polynomials can be represented by $\phi(L)$ and $\theta(L)$, respectively.

If the series is difference-stationary, the integrated autoregressive moving average (*ARIMA*) model

[1] The first discussion about this matter is provided by Theil (1954) and Grunfeld and Griliches (1960).

is implemented. The series Y_t is said to be $ARIMA(p,d,q)$ if

$$\phi(L)(1-L)^d Y_t = \theta(L)\varepsilon_t \tag{2}$$

where d is the d^{th} difference operator.

Empirical studies on macroeconomic forecasting, such as Stock and Watson (1999), found that including leading indicators into the model improves forecasting performance. Hamiltion an Perez-Quiros (1996) suggested that Composite Leading Indicator (CLI) is a good predictor for GDP especially in case of turning point in an economy. Since Thailand export depends heavily on trade-partners' economy, we believe that our forecasting performance will improve if we include CLI into the model. The *ARIMA* model will be extended into *ARIMA* model with explanatory variable (X), called $ARIMAX(p,d,q)$. Specifically, $ARIMAX(p,d,q)$ can be represented by

$$\phi(L)(1-L)^d Y_t = \Theta(L)X_t + \theta(L)\varepsilon_t$$

where X_t is trade partner's CLI. Moreover, some export commodities, such as rice and agricultural goods, have seasonal feature. We employ seasonal *ARIMA* and seasonal *ARIMAX* model to capture seasonality.

2.2 Methodology

In this section, we explain the method that we use to evaluate forecasting performance. We compare the out-of-sample performance between *ARIMA* model and *ARIMAX* model. Moreover, we compare the forecasts from direct and indirect method.

In our empirical study, we separate our data into two sets: estimation sub-sample and evaluation sub-sample. For each forecasting experiment, we use rolling window method to keep the length of estimation period constant. We perform the following steps for our experiments:

1. We employ Augmented Dickey-Fuller test (Dickey and Fuller, 1979, 1981) to our export series. We find that all our export data that categorized into countries or commodities have a unit root. Then, for both *ARIMA* and *ARIMAX* model, we fixed the order of integration (d) at one.

2. For each experiment, we employ Schwarz Information Criteria (SIC) to select the optimal lags for autoregressive and moving average polynomials (p,q). In our experiment, we limit maximum order at six months. For explanatory variable, composite leading indicator (CLI), we tried different lags

between two to eight months and found that lag of three provide the superior result. Therefore, we use CLI with lag of three in our $ARIMAX(p,1,q)$ model.

3. After obtaining the optimal lags, we compute three-period-ahead forecasts.

4. We use rolling window method and repeat steps one to three.

5. We obtain a set of forecasting data from both models. We compare accuracy of both models by using Mean Square Forecast Error (MSFE) and Diabold-Mariano test.

Diebold-Mariano test

Let Y_t be the data that we need to forecast. We have two forecasting series $Y^1_{t+h|t}$ and $Y^2_{t+h|t}$ which are based on the data until time t. Specifically , $Y^1_{t+h|t}$ is a forecasting series from an *ARIMA* model and $Y^2_{t+h|t}$ is a forecasting series from an *ARIMAX* model. Then we can compute forecast errors $\varepsilon^1_{t+h|t} = Y_{t+h} - Y^1_{t+h|t}$ and $\varepsilon^2_{t+h|t} = Y_{t+h} - Y^2_{t+h|t}$

Assume that we predict h-step forecasts for $t = t_0, ..., T$. Then, we have two series of T_0 forecasts and two errors series, $\{\varepsilon^1_{t+h|t}\}^T_{t_0}$ and $\{\varepsilon^2_{t+h|t}\}^T_{t_0}$. We can measure an accuracy using the loss function $L(Y_{t+h}, Y^i_{t+h|t}) = L(\varepsilon^i_{t+h|t})$ where $i = 1,2$. Two popular loss functions are square error loss $((\varepsilon^i_{t+h|t})^2)$ and absolute error loss $(|\varepsilon^i_{t+h|t}|)$. If any two forecast methods perform indifferently, these loss measures should be close. Diabold-Mariano test is based on the null hypothesis

$$H_0 : E[L(\varepsilon^1_{t+h|t})] = E[L(\varepsilon^2_{t+h|t})]$$

against

$$H_1 : E[L(\varepsilon^1_{t+h|t})] \neq E[L(\varepsilon^2_{t+h|t})].$$

Furthermore, we can rewrite these hypotheses in term of the difference between the losses, $E[d_t = 0]$ where $d_t = L(\varepsilon^1_{t+h|t}) - L(\varepsilon^2_{t+h|t})$. Under the null hypothesis that both forecasting methods provide the same accuracy, $H_0 : E(d_t) = 0$, the Diebold-Mariano (D-M) statistic can be calculated by

$$S = \frac{\bar{d}}{(\widehat{LRT_{\bar{d}/T}})^{1/2}}$$

where $\bar{d} = \frac{1}{T_0}\sum^T_{t=t_0} d_t$ and $LRV_{\bar{d}} = cov(d_t, d_{t-j})$. Diebold and Mariano (1996) has shown that under H_0 is true, $S \sim N(0,1)$. In practice, we will calculate the different between the loss function and regress the differences on a constant term, then we will test the significance of the constant term.

2.3 Data

The data considered in this study include monthly export data by countries and commodities from Thailand Ministry of Commerce and Composite Leading Indicator from OECD[2]. Ten principal countries and two groups i.e. China, Japan, USA, EU(27), Hong Kong, Malaysia, Singapore, Vietnam, Indonesia, Australia, other countries are considered in our study. Export by countries data are ranging from January 1996 to December 2011. We fixed an estimation period at 120 months, therefore we have 72 forecasts. For export by commodities, the data are ranging from January 2002 to December 2011. We fixed the estimation period at 72 months, so we have 60 forecasts. For CLI, there is no monthly CLI for Malaysia, Singapore, Indonesia, Vietnam, and other countries. Five Asian countries CLI is used for Malaysia, Singapore, Indonesia, Vietnam and OECDs CLI is used for other countries group.

3 Empirical Results

After we obtained the optimal lags for p and q, we forecast the export series and compare the forecasts from *ARIMA* model and *ARIMAX* model. The Mean Squares Forecast Errors ($MSFE$) and Diabole-Mariano ($D-M$) statistics are calculated and shown in Table 1. We consider one- to three-period-ahead forecasts. From Table 1, we find that the forecast performance of *ARIMAX* model are statistically superior than one of *ARIMA* model in case of exports to Japan, USA and EU countries for all forecast horizons we considered. Furthermore, for the rest of the world data and whole world, the *ARIMAX* model provide the better forecasting results. On the contrary, for exports to Hong Kong, Malaysia, Singapore, India, Vietnam, and Indonesia, the *ARIMAX* model mostly provides the better $MSFE$ than the *ARIMA* model. However, those $MSFEs$ are not statistically different.

In order to address our second question, we consider two methods of forecasting,i.e. direct and indirect methods. In this exercise, we select four major trade counterparts which are China, Japan, USA and EU (27 countries) to compare these two forecast procedures. We also consider both *ARIMA* and *ARIMAX* models. The results are presented in Table 2. We found that, except for export to Japan data with *ARIMA* model, the forecasting performances of direct and indirect approaches are not statistically different.

Moreover, we consider the usefulness of CLI in forecasting exports in each commodity group. The results are presented in Table 3. We found that mostly the performances of *ARIMA* and *ARIMAX* models for

[2] `stats.oecd.org/Index.aspx?DatasetCode=MEI_CLI`

Table 1: MSFEs - Export by countries, using ARIMA and ARIMAX

	$h=t+1$			$h=t+2$			$h=t+3$		
	ARIMAX	*ARIMA*	$D-M$	*ARIMAX*	*ARIMA*	$D-M$	*ARIMAX*	*ARIMA*	$D-M$
China	37,815	46,363	-8,547	48,704	66,521	-17,816	58,852	78,364	-19,512
Japan	17,848	26,249	-8,401.4**	22,977	37,931	-14,953***	31,399	55,794	-24,395***
USA	13,261	22,363	-9,102.6***	18,508	32,063	-13,555***	21,630	43,152	-21,522***
EU(27)	27,801	43,958	-16,156.8**	34,633	64,565	-29,932**	40,807	81,851	-41,043**
Hong Kong	28,391	29,452	-1,061	35,823	43,045	-7,222	40,635	39,095	1,539
Malaysia	18,689	20,185	-1,497	23,117	27,237	-4,119	28,310	33,131	-4,820
Singapore	22,500	24,574	-2,074	26,772	32,005	-5,233	32,606	42,032	-9,425**
India	2,430	2,346	84	3,082	2,962	120	3,387	3,334	52
Vietnam	5,545	5,838	-292.3	6,485	7,309	-824	7,405	8,341	-935
Indonesia	7,495	9,665	-2,170	8,675	11,355	-2,680	11,500	15,128	-3,628
Australia	35,772	31,962	3,809*	44,462	42,203	2,259	48,433	46,229	2,203
Other countries	163,921	226,126	-62,205**	208,139	291,066	-82,927**	235,264	330,022	-94,758**
World	1,834,572	2,891,132	-1,056,560**	2,380,657	4,296,280	-1,915,623***	3,000,026	5,183,303	-2,183,278**

Source: Authors' calculation from forecasts using data between January 1996 and December 2011 with 120-month rolling window.
Note: Numbers in column D-M are the difference between errors from two models. Negative value means the MSFE from *ARIMAX* model is less than one from *ARIMA* model. The differences marked with an asterisk (***) are significant at the 1 percent level of significance; (**), at the 5 percent level; and (*), at the 10 percent level, respectively.

Table 2: MSFEs - Export by countries, using Indirect and Direct procedures

	$h=t+1$			$h=t+2$			$h=t+3$		
	Indirect	Direct	D-M stat	Indirect	Direct	D-M stat	Indirect	Direct	D-M stat
China									
ARIMAX	43,885	43,880	-5	76,273	68,390	-7,882	114,440	92,195	-22,234
ARIMA	59,638	54,994	-4,643	96,874	90,556	-6,317	123,998	107,264	-16,734
Japan									
ARIMAX	22,241	19,817	-2,424	34,044	35,254	1,210	51,326	47,694	-3,632
ARIMA	26,572	35,029	8,457**	41,887	58,468	16,581***	62,049	86,570	24,520***
USA									
ARIMAX	20,383	19,287	-1,096	27,149	27,602	452	42,359	38,733	-3625
ARIMA	28,769	28,295	-473	43,050	45,059	2,008	58,815	68,817	10,001
EU(27)									
ARIMAX	36,767	34,080	-2687	51,626	57,964	6,337	71,923	67,925	-3,997
ARIMA	57,747	57,818	71	88,982	91,878	2,896	117,265	128,288	11,023

Source: Authors' calculation from forecasts using data between January 2002 and December 2011 with 72-month rolling window.
Note: Numbers in column D-M are the difference between errors from two models. Negative value means the MSFE from indirect method is less than one from direct method. The differences marked with an asterisk (***) are significant at the 1 percent level of significance; (**), at the 5 percent level; and (*), at the 10 percent level, respectively.

export commodities in China, Japan, U.S. and E.U. (27 countries) are not statistically different. Moreover, in case that both models provide different performance, there is uncertainty about which model is better.

Table 3: MSFEs - Export to USA, Japan, China, EU(27) by commodities, using *ARIMA* and *ARIMAX* models

	USA			Japan			China			EU(27)		
Product	*ARIMAX*	*ARIMA*	D-M stat	*ARIMAX*	*ARIMA*	DM	*ARIMAX*	*ARIMA*	DM	*ARIMAX*	*ARIMA*	DM
Rice	334	306	27	–			573	155	418	71	49	22*
Rubber	219	278	-59*	521	951	-429***	4,242	5,312	-1070	313	433	-120
Rubber products	76	88	-11	37	22	15*	416	549	-133	62	69	-7
Cassava products	1.2	0.8	0.3	13	14	-0.8	1,004	1,042	-38	53	43	10
Frozen shrimp	200	162	38	17	18	-0.9	2.3	2.8	-0.5	37	23	13**
Canned seafood	184	144	39*	62	53	9**	–			111	118	-7
Canned fruit	46	49	-3	4.6	4.2	0.4	0.32	0.30	0.01	39	40	-0.8
Sugar	–			302	242	60	–			–		
Computer and parts	2,094	2,072	21	500	556	-56	6,359	7,374	-1,015	2,349	2,698	-349
Motor cars and parts	27	33	-6	1,057	1,195	-138	26	24	2.5	594	911	-316*
Electrical apparatus and parts	360	403	-43	272	257	14	129	107	21	286	375	-89**
Plastic and plastic products	26.5	26.0	0.5	168	224	-56	497	614	-117*	60	96	-36*
Jewelry and accessories	365	466	-101	94	92	2	7.1	6.4	0.7	564	683	-119
Garment	139	241	-101**	47	49	-2	26.2	28.6	-2.4	10,725	10,389	336
Other products	6,064	9,745	-3681**	8,753	9,130	-376	11,394	10,345	1,049	20,087	20,863	-776
Total export	19,287	28,295	-9008*	19,817	35,029	-14,211***	43,880	59,638	-15,757	34,080	57,818	-23,738*

Source: Authors' calculation from forecasts using data between January 2002 and December 2011 with 72-month rolling window.
Note: Numbers in column D-M are the difference between errors from two models. Negative value means the MSFE from *ARIMAX* model is less than one from *ARIMA* model. The differences marked with an asterisk (***) are significant at the 1 percent level of significance; (**), at the 5 percent level; and (*), at the 10 percent level, respectively. "–" indicates that data is too few to implement time series models.

4 Conclusion

This paper has examined the forecasting performance of *ARIMAX* and *ARIMA* models for Thailand export data. We also examined the direct and indirect approaches of forecasting. We find that, for country-level data, *ARIMAX* model outperforms *ARIMA* model only in some principal trade partners; Japan, USA, EU(27) and Australia. For commodity-level data, mostly, there is no statistically difference between performances of *ARIMAX* and *ARIMA* model. Moreover, for Thailand export data, indirect method does not provide better forecasting performance than direct method. As a result, we suggest practitioners at the Ministry to implement the *ARIMAX* model for export data using the direct method.

References

[1] Box and Jenkins (1970) *Time Series Analysis: Forecast and Control*. San Francisco: Holden-Day.

[2] Diebold, F.X. and Mariano, R.S. (1995). "Comparing predictive accuracy," *Journal of Business and Economic Statistics*. 13, 253 – 263.

[3] Grunfeld, Y., and Griliches, Z. (1960). "Is aggregation necessarily bad?" *The Review of Economics and Statistics*. 42, 1 – 13.

[4] Hamilton J.D. and G. Perez-Quiros (1996) "What Do the Leading Indicators Lead?," *The Journal of Business*, 69, 27 – 49.

[5] Hubrich, Kirstin. (2005). "Forecasting euro area inflation: Does aggregating forecasts by HICP component improve forecast accuracy?" *International Journal of Forecasting*. 21, 119 – 136.

[6] Jantarakolica, T. and P. Chalermsook. (2012). "Test Forecast Performance using Leading Indicator: A Case Study of Thai Export," Working paper, Thammasat University.

[7] Keck, A., A. Raubold, and A. Truppia. (2009). “Forecasting International Trade: A Time Series Approach,” *OECD Journal: Journal of Business Cycle Measurement and Analysis*, 9(2009), 157–176.

[8] Makridakis S., S. Wheelwright, and R.J. Hyndman (1998) *Forecasting: methods and applications*, Wiley: New York

[9] Stock, J.H. and M.W. Watson (2003) *Introduction to Econometrics*. Addison Wesley.

[10] Stock, J.H. and M.W. Watson (1999) “Forecasting Inflation.” Journal of Monetary Economics. 44 (1999), 293–335.

[11] Theil, H. (1954). *Linear Aggregation of Economic Relations*. Amsterdam: North Holland.