Understanding Growth-at-Risk: A Markov-Switching Approach*

Dario Caldara† Danilo Cascaldi-Garcia‡

Pablo Cuba-Borda§ Francesca Loria¶

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Abstract

We show that a Markov-switching model with endogenous transition probabilities can replicate a common finding in the Growth-at-Risk literature, that is that the (conditional) mean and volatility of future growth are negatively correlated. The model also provides an intuitive interpretation of macroeconomic risk: (endogenous) regime uncertainty generates tail risk. The higher the regime uncertainty, the starker are the differences in the growth outlook between a normal and a bad state of the economy. The model is a new tool to assess the risk of tail events, such as recessions, and to evaluate the likelihood of point forecasts. We also propose real-time measures of financial conditions and economic activity for the United States and use these measures to construct conditional quantiles and predictive distributions of average GDP growth over the next 12 months. We find that periods of high macroeconomic and financial distress, such as the Global Financial Crisis and the COVID-19 pandemic, are associated with low average future growth, high uncertainty, and risks tilted to the downside.

Keywords: Growth-at-Risk, Regime-Switching Regression, Quantile Regression
JEL codes: C53, E23, E27, E32, E44

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†Federal Reserve Board, International Finance Division, Washington-DC. dario.caldara@frb.gov.

‡Federal Reserve Board, International Finance Division, Washington-DC. danilo.cascaldi-garcia@frb.gov.

§Federal Reserve Board, International Finance Division, Washington-DC. pablo.a.cubaborda@frb.gov.

¶Federal Reserve Board, Research & Statistics Division, Washington-DC. francesca.loria@frb.gov.

> *"While the economic response has been both timely and appropriately large, it may not be the final chapter, given that the path ahead is both highly uncertain and subject to significant downside risks."*
>
> Jerome H. Powell
> May 13, 2020.

1 Introduction

Aggregate economic activity features persistent swings in average GDP growth and its volatility. These fluctuations often involve periods of above-average growth and low variance associated with expansions, such as the U.S. Great Moderation during the 1990s and part of the 2000s. Periods of expansion are interrupted by periods of declining average growth and higher volatility, such as the 1970s and 1980s, or the sharp contractions in economic activity during the Great Recession of 2008-2009. The systematic relation between average growth and volatility is central for understanding fluctuations in aggregate consumption and welfare (Barro, 2009; Nakamura *et al.*, 2017), time variation in risk premia (Gourio, 2012), asset price volatility (Schorfheide *et al.*, 2018), and as we show in this paper, constitutes the foundation behind the growth-at-risk framework introduced by Adrian *et al.* (2019).

Understanding the relation between mean and volatility of future GDP growth is central for the design of macroeconomic and macro-prudential policies. Risk assessment considerations are at the forefront in central banks, and the quantification of risks is particularly valuable at the onset of turning points in the economic cycle. However, forecasting economic activity is exceptionally difficult even within short horizons, and understanding the driving forces and the transmission of tail risk remains an open question.

In this paper, we offer a unifying framework for understanding tail risks by quantifying the connection between economic conditions and the time variation in the mean and volatility of expected future growth. We estimate a Markov-switching model in which the transition probabilities depend on real-time indicators capturing financial and macroeconomics conditions, implying that such conditions exert a nonlinear effect on future GDP growth. We show that our model with discrete regime changes provides a plausible characterization of tail risk that takes the form of transitions between regimes of high-growth-low-volatility and low-growth-high-volatility. An essential advantage of our approach is that the Markov-switching environment sheds light on the features of the data by identifying the determinants and time variation in the conditional distribution of future GDP growth. We, therefore, provide a natural interpretation for the results obtained using quantile regressions (Adrian *et al.*, 2019; Plagborg-Møller *et al.*, 2020).

We track the evolution of macroeconomic and financial conditions by building comprehensive and timely indicators. In particular, we use dynamic factor models, similar to Aruoba *et al.* (2009) and Banbura *et al.* (2013), to construct real-time measures of financial and economic activity that summarize the information available at weekly, monthly, and quarterly frequency. A direct by-product of our procedure is a monthly estimate of real GDP growth, that adds up to the official quarterly data, but is available in real-time.

We highlight three main results. First, our analysis emphasizes the direct connection between the conditional distribution of future GDP growth and the evolution of our estimated financial and macroeconomic factors. We first present a simplified Markov-switching model in which we show the intuition that connects tail-risk with time-variation in the sensitivity of future growth to underlying financial and macroeconomic conditions. The basic intuition is that the mean and volatility of expected growth negatively co-move in expansions and recessions. In our model, financial and macroeconomic conditions also affect growth indirectly through the sensitivity of future GDP growth and transition probabilities of changing regimes.

Second, our model provides a unifying framework to understand and assess tail risk. We extend our simple Markov-switching model to allow three states and two independent Markov chains that drive the dynamics of mean and volatility in the conditional distribution of future

GDP growth. We show that this model produces quantitatively similar results to the more flexible approach of Quantile Regressions of Adrian *et al.* (2019). More importantly, we show that when applied to U.S. data, both the Markov-switching and quantile regression models agree upon the interpretation of the period of tail risk and their determinants. This result supports the notion that estimates of tail risk have robust features that can be captured with multiple models and in real-time.

We view the Markov-switching and quantile regression models as complementary tools of an arsenal of models to quantify risks. On the one hand, the Markov-switching model provides more structure and allows us to attach an interpretation closely connected to the endogenous regime changes generated by nonlinear structural models. On the other hand, the quantile regression model is a more flexible system that lets the data drive the results, but the interpretation of estimated quantiles might be cumbersome. Although our framework allows us to draw a clear connection between models, we recognize that there are periods in which Markov-switching and quantile regression models might disagree. For example, the Markov-switching model characterizes the period associated with the debt ceiling debate and fiscal-cliff in 2001 as an episode of lower growth but average volatility. In contrast, the quantile regression model uncovered an increase in downside risk. For this reason, we believe that a single tool is not enough to inform policy decisions.

Third, when risks of adverse outcomes are elevated relative to historical experience, risk management considerations might require policy actions even if the point forecasts from tail risk models turn inaccurate. This risk management approach is central to policy-making institutions using a wide range of indicators and various models to assess tail risk. The Markov-switching and quantile regression models offer additional tools for risk assessment that provide valuable input for decision-makers.

We illustrate the trade-off between forecast-accuracy and risk assessment by examining the early stages of the global financial crisis and the build-up of risk during the COVID-19 shock. Our models provide a correct characterization of the downside risk to the outlook in both episodes, despite significant pseudo-out-of-sample forecast errors concerning the median projections. Our take is that a policymaker using tools to model tail risk has a better assessment of the potential economic downturn.

Related literature. Our contribution to the literature on tail risks builds on the work of Adrian *et al.* (2019) that uses quantile regressions, a very flexible tool to capture time variation in tail risk. A central result in their analysis is that downside risks to GDP growth in the U.S. vary notably over the cycle, a feature consistent with models featuring financial frictions and occasionally binding credit constraints—see, for example, Fernández-Villaverde *et al.* (2019) and Jensen *et al.* (2020). However, there is little understanding of the relationship between quantile regressions and other approaches typically employed in macroeconomics. We offer a unifying framework for understanding tail risks, by providing the connection between quantile regressions, Markov-switching models, and nonlinear dynamic general equilibrium models.

Our Markov-Switching specification captures the time-varying sensitivity of endogenous variables to the underlying state of the economy typically present in nonlinear models. In the presence of binding constraints that lead to equilibrium multiplicity as in Aruoba *et al.* (2018) or Gertler *et al.* (2019), our regime-switching specification is well suited to capture the sharp changes in the sensitivity of GDP growth to the state of the economy across regimes. From a modeling perspective, endogenous transition probabilities resemble the solution in models with occasionally binding constraints in which the resulting decision rules have a two-regime representation in which the switching across regimes is triggered endogenously by the state variables of the model (Aruoba *et al.*, 2020; Benigno *et al.*, 2020; Lansing, 2019).

Our dynamic factor model shares similar advantages to Schorfheide and Song (2020) that use a mixed-frequency Vector Auto Regression (VAR). We can also estimate the model even in the presence of large swings in the data using missing observations. In periods of extreme volatility, we censor the data for parameter estimation, but we do not fully account for the changes in volatility as in Lenza and Primiceri (2020). In a second step, taking the estimated parameters as given, our macroeconomic and financial factors make use of all available data. Therefore we do not fully discard influential observations as in Schorfheide and Song (2020) and Primiceri and Tambalotti (2020).

The rest of the paper is organized as follows. Section 2 focuses on the estimation of financial and macroeconomic factors. Section 3 lays out the intuition behind our empirical approach using a simplified Markov-switching model. Section 4 presents our benchmark model and compares results with the quantile regression framework. Section 5 shows the application of our models for risk assessment. Section 6 summarizes robustness specification and results. Section 7 concludes.

2 Economic and Financial Conditions in Real-Time

We build indicators that are able to monitor financial and macroeconomic conditions in a timely manner. We start by briefly stating the econometric procedure to estimate a Dynamic Factor Model (DFM) for monthly observations. Instead of looking at a large set of variables, we focus on a parsimonious set of series that is readily available and that summarizes the activity of key parts of the financial and real structure of the economy.

A direct by-product of our procedure is a monthly estimate of real GDP growth, that adds up to the official quarterly version, but is available in real-time. We show that our GDP series, together with the financial and macroeconomic factor gives some hints about the features that characterize growth-at-risk. Our monthly GDP estimates, together with the financial and macroeconomic factors obtained from the DFM, will be the inputs for the nonlinear models that we study in Sections 3 and 4.

2.1 A Dynamic Factor Model for Monthly Observations

The use of statistical models to summarize a large number of economic time series has a long-standing history in econometrics. In the macroeconomics realm, dynamic factor models have played a prominent role in summarizing the sources of economic fluctuations and constitute a widely used framework for real-time macroeconomic monitoring and forecasting (**?**Kose *et al.*, 2003; Giannone *et al.*, 2008; Aruoba *et al.*, 2009). We build on this tradition and estimate a dynamic factor model that summarizes both macroeconomic and financial conditions in real-time.

We start describing the structure of a generic DFM that can be applied to monthly data and we later discuss the issues of asynchronous data releases and missing observations. Let Y_t denote a $n \times 1$ vector of observed data. An element of the vector Y_t is denoted $y_{i,t}$ and we postulate that each element of $y_{i,t}$ is related to a common unobserved factor, x_t, and to an idiosyncratic component, $\varepsilon_{i,t}$, through the following set of equations:

$$\begin{aligned} y_t^i &= a_i + \lambda_i x_t + \varepsilon_{i,t}, \quad i = 1, \dots, n, \\ \varepsilon_t^i &= \gamma_{i,1}\varepsilon_{i,t-1} + \cdots + \gamma_{i,p}\varepsilon_{i,t-p} + \nu_{i,t}, \\ x_t &= \phi_1 x_{t-1} + \cdots + \phi_q x_{t-q} + \eta_t, \end{aligned} \tag{1}$$

where a_i is a constant term and λ_i is the coefficient that measures the loading of the $i-th$ series to the common factor. The evolution of the common factor and of the idiosyncratic component are governed by the auto-regressive process of orders p and q, respectively. The term $\nu_{i,t}$ represent pure noise components associated to the idiosyncratic factors for each observable variable and are drawn independently from each other from the distributions $\mathcal{N}(0, \sigma_i^2)$. Similarly, the term η_t is the stochastic component of the common factor and is drawn independently from the $\mathcal{N}(0, \sigma_\eta^2)$ distribution. For our macroeconomic factor we set $p = 3$, $q = 1$. While for the estimation of the financial factor we set $p = q = 1$.

A challenge with macroeconomic data releases is the "ragged-edge" pattern, meaning that the release schedule is scattered, and data availability increases at different moments of a typical quarter.[1] A related challenge is that of handling missing observations. Traditional statistical approaches for summarizing many indicators, such as principal components, demand a balanced panel. In this case, it would mean truncating the information set to the point where all data is available, for example in March, GDP data for the first quarter is not available, which would force us truncate data through December of the previous year to balance the panel. However, if one wants to understand rapid changes in the economic outlook, discarding more recent data is undesirable.

Our DFM approach takes advantage of an expectation step[2] fills the missing gaps. Once the actual data is released, the 'expected' value is replaced, and the model re-optimizes. The advantage of such a procedure is the ability to use all data available. While GDP data is the most informative about the outlook, the fact that it is available only at a quarterly frequency makes it hard to reconvene its signals with more timely and frequent variables. Usual statistical approaches that do not deal with mixed frequencies need to either disregard the GDP information if using monthly data, losing an essential component of the outlook or aggregate monthly data into quarterly, losing the rich information of short-term swings captured during the quarter.

Our DFM approach deals with the mixed frequency problem by treating the 'unobserved monthly GDP' as a latent variable and uses quarterly GDP information only when available. Formally, following Aruoba *et al.* (2010), if the signal y_t^i is observed at a quarterly frequency,

[1] In Appendix A we show an example of the typical data flow of a macroeconomic information set.

[2] See Bańbura and Modugno (2014) for details on the Expectation-Maximization algorithm applied to dynamic factor models.

such as is the case with GDP growth data, then we use the following approximation:

$$y_t^i = \begin{cases} 3c^i + \lambda^i(x_t + x_{t-1} + x_{t-2}) + \varepsilon_{i,t} + \varepsilon_{i,t-1} + \varepsilon_{i,t-2}, & \text{if data is observed} \\ NA, & \text{otherwise} \end{cases} \tag{2}$$

where λ_i is the coefficient that measures the loading of the latent variable x_t (and its lags) to GDP, and the latent variable x_t represents a real-time estimate of a monthly GDP series as a direct byproduct of this approach.

2.2 Underlying Data

We summarize financial and macroeconomic conditions with two representative factors. We focus on achieving factors that accurately capture the signals from different segments of the economy and, at the same time, are sufficiently timely, allowing for a real-time follow up of the outlook developments. We opted for a parsimonious approach to estimating our factors with small-scale DFMs based on a few selected representative variables, instead of including several variables that may bring different conflicting signals. The inclusion of a large number of potentially correlated variables can lead to coefficient estimates that are not precisely identified and spurious, especially when it comes to non-linear dynamics that happen only in specific regimes. Our results are not conditional on such a parsimonious approach, being robust to several alternative macroeconomic and financial trackers available in the literature (presented in detail in Appendix D).

For the macroeconomic factor, we include monthly observations of industrial production (IP), retail sales (RS), the new export order component of the Markit's Purchasing Managers Index (PMI), and initial claims for unemployment insurance (UCLAIMS). In addition to monthly time series, we include GDP growth as an additional observable. IP and RS are expressed in monthly annualized growth rates, PMI is reported as a diffusion index, and

UCLAIMS are expressed as a share of the labor force.

For the financial factor, we use a separate DFM using monthly financial variables. Our information set includes a parsimonious set of representative variables that capture not only credit and financial conditions, but also non-financial vulnerabilities and macro financial imbalances. Specifically, we include monthly observations of the VXO (a volatility index of the S&P 100), the excess bond premium (EBP, Gilchrist and Zakrajšek, 2012, and Favara *et al.*, 2016), the TED spread (the difference between the 3-month LIBOR rate and the 3-month Treasury bill) and the CBILL spread (the difference between the 3-month financial-commercial paper rate and the 3-month Treasury bill). Monthly GDP, macroeconomic and financial factors are estimated from January 1973 to May 2020. Table A-1 in the Appendix provides more information on the macroeconomic and financial data sources and availability.

2.3 Macroeconomic and Financial Factors

Our macroeconomic factor accurately tracks the GDP evolution and periods of macroeconomic downturns, such as NBER identified recessions. Figure 1a shows our estimated common macroeconomic factor (m_t), and the underlying (normalized) data. For ease of comparison, all series are transformed into standardized units and low-frequency trends removed using a bi-weight filter as in Stock and Watson (2016). The macroeconomic factor is less volatile than the underlying data, but it can capture the common patterns in macroeconomic variables. The macroeconomic factor correlates with the underlying data, in particular, the correlation is above 70% for monthly indicators as well as for the observed quarterly GDP.[3]

While each underlying financial variable has its dynamics, the financial factor seems to capture their main common movements. Figure 1b presents the estimated common financial

[2] See Adrian *et al.* (2015) and Aikman *et al.* (2020) for an extensive analysis of the implications of nonfinancial-sector credit on financial conditions and monetary policy.

[3] Appendix A shows detailed correlations of the financial and macroeconomic factors with the underlying data series.

factor (f_t) and the underlying (normalized) data. The financial factor spikes in periods of recessions (NBER identified), and on well-known financial distress episodes, as around the Black Monday (1987) and the dot.com bubble (the early 2000s). The contemporaneous correlations of the underlying variables and the financial factor, ranges from 0.78 with the CBILL spread, to 0.84 with the EBP, confirming the strong relationship captured by the financial factor.

Our estimated monthly GDP series adds to the official quarterly GDP growth rate and is comparable to other alternatives available in the literature.[4] As this series is a direct construct from the macro factor, it gives a more timely indication of the economic situation than the delayed official GDP. Figure 1c presents our monthly GDP (year over year) with three alternative measures: the Weekly Economic Index (WEI, Lewis *et al.*, 2020), the IHS Markit MGDP (IHS Markit website), and the Stock and Watson series (S-W, Stock and Watson, 1989). Our measure is a smoothed version that perfectly aligns with the more volatile S-W and IMGDP. The WEI series is also smoother than the MGDP and the S-W series. The WEI series also overshoots the 2010/2011 recovery and the 2015 slowdown, a period in which our monthly GDP series coincides with the MGDP series. Over the pairwise common samples our measure correlates strongly with the S-W series(0.94), with the MGDP (0.89), and with the WEI series (0.82) .

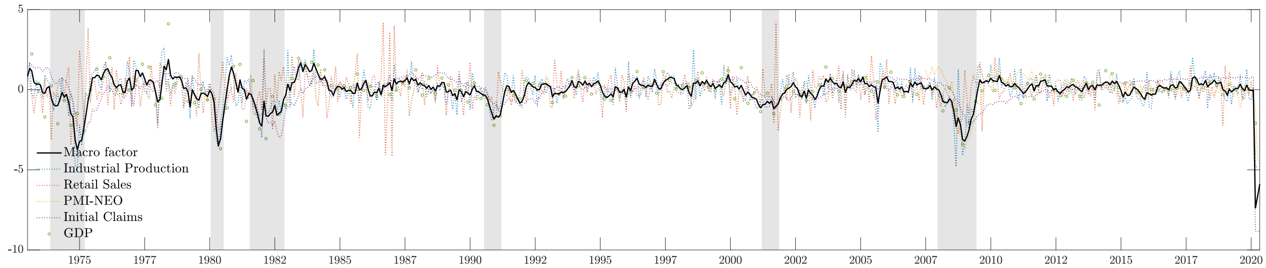
Finally, our monthly GDP and the macroeconomic and financial factors give some hints about the existence of (at least) two identifiable business cycle regimes: one characterized by periods of low growth and high volatility, and another of high growth and low volatility. Figure 1d presents a rolling window of 12 months of the mean and standard deviation of the monthly GDP.[5] Recession periods, with negative average monthly GDP growth, periods of macroeconomic downturn, captured by negative swings in the average of the macroeconomic

[4]Figure A-1 in the Appendix presents the estimated monthly GDP at the annualized rate and the official quarterly GDP growth.

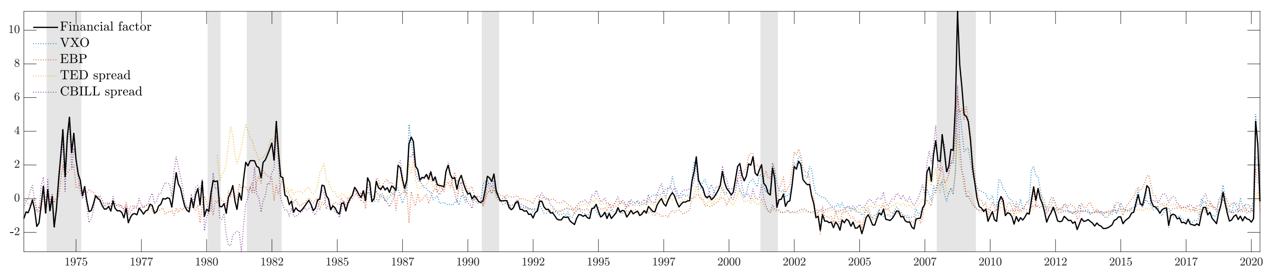
[5]Figure A-2 in the Appendix presents similar exhibits for the macroeconomic and financial factors.

factor, and periods of financial distress, characterized by increases in the average financial factor, are directly accompanied by increased volatility in each of these indicators. The rolling mean and standard deviation share a correlation of -0.19 for the monthly GDP, -0.58 for the macroeconomic factor and 0.75 for the financial factor. A potential third regime of high GDP growth and high volatility seems to dictate the 1970s dynamics. If we restrict our analysis to 1980 onward, the correlation between the mean and standard deviation of monthly GDP increases to -0.40. We formally explore the existence and how to model these potential regimes in the next Section.

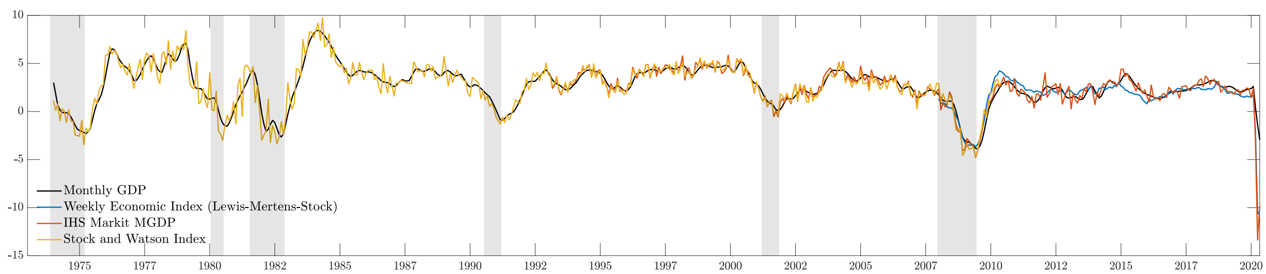
Figure 1: Macroeconomic and Financial Factors and Monthly GDP.



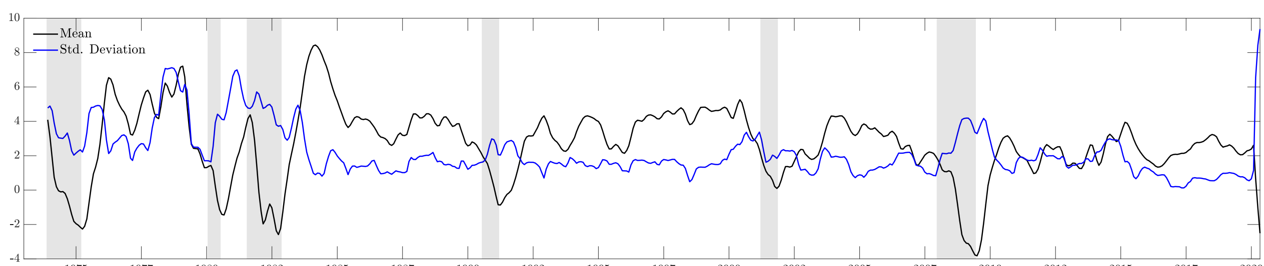
(a) Macroeconomic Factor and Underlying Data.



(b) Financial Factor and Underlying Data.



(c) Alternative Measures of Monthly GDP (year over year).



(d) Mean and Standard Deviation of GDP Growth (12-month Rolling Window).

Note: Macroeconomic and Financial factors and Monthly GDP constructed using a Dynamic Factor Model as described in Section 2.1, and underlying data as described in Section 2.2. Alternative measures of monthly GDP are the Weekly Economic Index (Lewis *et al.*, 2020, available from January 2008), the IHS Markit MGDP (IHS Markit website, available from January 1992), and the Stock and Watson Index (Mark Watson's website, discontinued, available until June 2010). Shaded areas denote NBER dated recession months.

3 A Regime-Switching Growth-at-Risk Model

Our analysis emphasizes the connection between the conditional distribution of future GDP growth and the evolution of observable series, such as our macroeconomic and financial condition indicators. In this section, we offer a semi-structural econometric model to help make this connection. Our specification is motivated by the observation that models that can match the interaction between real variables, such as investment and output, with financial variables, such as asset prices or bank's leverage, build on nonlinear frameworks that can capture the interaction of time-variation in risk and macroeconomic outcomes (Gourio, 2012; Bocola, 2016; Fernández-Villaverde *et al.*, 2019).

Several mechanisms exist to generate nonlinear relations in aggregate economic activity. On the one hand, the literature on financial frictions has emphasized the connection between households, firms, and bank leverage as a critical determinant of such nonlinear relation (Jordà *et al.*, 2017; Gertler *et al.*, 2019). On the other hand, the origin of these nonlinearities can arise from the interactions of real frictions or constraints on fiscal and monetary policy, such as the zero lower bound (Fernández-Villaverde *et al.*, 2015; Aruoba *et al.*, 2018).

We capture these multiple mechanisms by taking a semi-structural approach that links future GDP growth to macroeconomic and financial conditions. Formally, we postulate a general model under the form:

$$\bar{\Delta} y_{t+1,t+12} = \alpha_y(s_t(f_t,m_t)) + \beta_y(s_t(f_t,m_t))f_t + \gamma_y(s_t(f_t,m_t))m_t + \sigma_y(s_t(f_t,m_t))\varepsilon_t^y, \quad (3)$$

$$f_t = \alpha_f + \beta_f f_{t-1} + \gamma_f m_t + \eta_f m_{t-1} + \sigma_f \varepsilon_t^f, \quad (4)$$

$$m_t = \alpha_m + \beta_m f_{t-1} + \eta_m m_{t-1} + \sigma_m \varepsilon_t^m, \quad (5)$$

Where $\bar{\Delta} y_{t+1,t+12}$ is average GDP growth over the next 12 months, f_t is the financial factor, m_t is the macroeconomic factor, $s_t(f_t, m_t)$ is a latent variable that depends on f_t and m_t, and where ε_t^y, ε_t^f and ε_t^m are $\mathcal{N}(0,1)$ i.i.d. shocks.

Equation (3) shows that average expected future GDP growth over the next 12-months is a function of the financial f_t and macroeconomic m_t factors, capturing the direct effect of our measures of economic activity on future outcomes. Also, we allow the "elasticity" of future GDP growth to be time-varying in response to the evolution of the macroeconomic and financial factors. Equations (4) and (5) specify the evolution and the feedback between the financial and macroeconomic factors.

From a modeling perspective, we assume that the evolution of the unobserved state variables of the system exerts a non-linear effect on the observed variable's response, in our case, future GDP growth. This specification resembles the decision rules in non-linear models, featuring the time-varying elasticity of control variables as a function of the economy's underlying state. Moreover, if the non-linearity takes the form of occasionally binding constraints or equilibrium multiplicity, we expect this sensitivity to change sharply as the economy transitions across regimes (Aruoba *et al.*, 2018; Lansing, 2019; Gertler *et al.*, 2019).

More generally, the time-varying sensitivity of future GDP growth to macroeconomic and financial conditions captures the essential features of what we consider to be Growth-at-Risk. Our flexible structure allows oscillations between periods associated with higher expected growth—such as the 2017-2019 period characterized by robust labor market developments in the U.S.—vs. Periods of low expected growth when macroeconomic and financial conditions deteriorate, such as during the Great Financial Crisis of 2007-2008 or the COVID-19 shock.

3.1 Endogenous Transition Model

We begin our analysis with a simple model in which the time-varying elasticity of future growth in response to macroeconomic and financial conditions are governed by a latent variable. In equation (3), the latent variable $s_t(f_t, m_t)$ follows a two-state discrete Markov stochastic process defined as $p_{ij} \equiv Pr(s_{t+1} = j | s_t = i)$, $\sum_{j=1}^{2} p_{ij} = 1$, $\forall i, j \in \{1, 2\}$, whose transition probabilities depend on the financial factor f_t and the macroeconomic factor m_t through a logistic function:

$$p_{12} = \frac{1}{1 + exp(a_{12} - b_{12} f_t - c_{12} m_t)}, \tag{6}$$

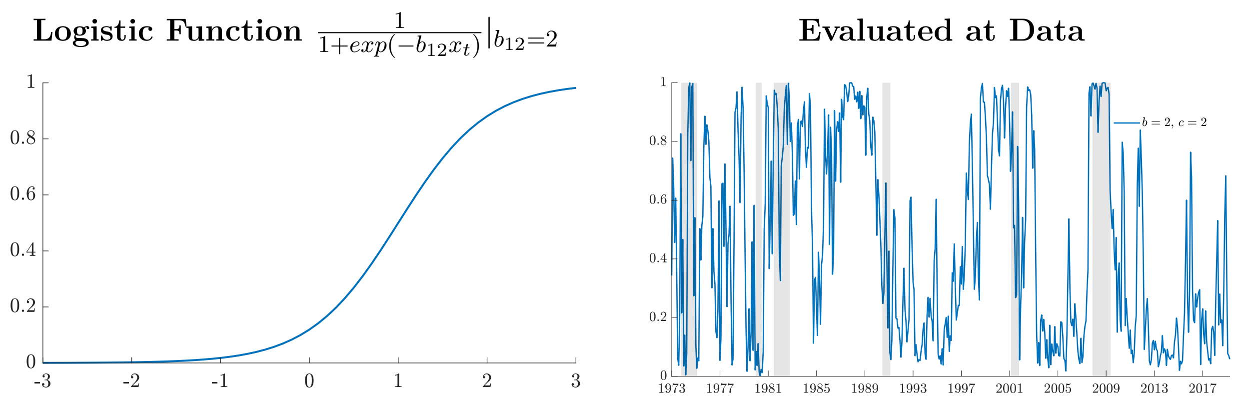
$$p_{21} = \frac{1}{1 + exp(a_{21} - b_{21} f_t - c_{21} m_t)}. \tag{7}$$

We associate the regimes in the Markov chain with periods of favorable economic conditions ($s_t = 1$), or a "normal" regime, and periods of adverse economic conditions ($s_t = 2$), a "bad" regime. The dependence of the transition probabilities on the macroeconomic and financial factors captures the fact that the likelihood of a downturn in economic activity increases when financial and macroeconomic conditions deteriorate. However, neither in isolation might be sufficient to trigger a full-blown recession. Moreover, we allow the transition from bad-to-normal and normal-to-bad regimes to respond asymmetrically to macroeconomic and financial conditions. This modeling strategy is related to the underlying structure of dynamic models featuring endogenous switching in the presence of occasionally binding constraints (Aruoba *et al.*, 2020; Benigno *et al.*, 2020; Fernández-Villaverde *et al.*, 2019).

This simple transition process is already able to capture well-known periods of economic distress. The left panel in figure 2 illustrates the transition probability from the "normal" to the "bad" regime for a hypothetical case in which the transition probability depends only on a generic variable, x_t. This plot shows that when the coefficient b_{12} is positive, the probability of transitioning from the "normal" to the "bad" regime increases when x_t increases. For example, if we associate x_t with the financial factor, deterioration of financial conditions increases the probability of transitioning to the "bad" regime. The right panel in figure 2 plots the transition probability for hypothetical values $b_{12} = c_{12} = 2$, and where we plug the observed time series for the financial and macroeconomic factors estimated for the U.S.

economy. The associated time series of transition probabilities increase substantially around periods associated with U.S. recessions depicted by the shaded areas. However, we also observe periods in which the transition probability signals a shift in a regime not associated with a recessionary episode. For example, in the period 1986-1989, we observed increasing financial stress but stable macroeconomic conditions.

Figure 2: Illustration of Endogenous Transition Probabilities.



NOTE: Illustration of endogenous transition probability. Left panel: Logistic function $\frac{1}{1+exp(-bx_t)}$ evaluated at $b = 2$. Right panel: Logistic function $\frac{1}{1+exp(-bf_t-cm_t)}$ evaluated with data on f_t and m_t at ($b = 2$, $c = 2$).

3.2 Estimation

We estimate the Markov Chain system using our monthly measures of GDP, macroeconomic, and financial conditions. We follow the Markov-Switching (MS, henceforth) perturbation approach described in Maih (2015) to estimate the model described by equations 3 to 5. In all our estimations, we construct expected future GDP growth as the arithmetic average over the subsequent twelve months of our estimated monthly GDP series. Hence, the variable of interest is the expected year-over-year change of monthly GDP at an annual rate. Before proceeding with estimation, we de-trend future GDP growth using a using average future growth over a 10-year rolling window.

We also provide measures of uncertainty around estimated parameters. We employ an MCMC sampler with 20,000 draws to jointly draw from the posterior distribution of parameters and transition probabilities of the model. We report all statistics using 2,000 equally spaced draws from the posterior distribution. The regression runs from January 1973 to May 2019, the last available date for average GDP growth over the next 12 months. Appendix B provides details on model estimation, hyperparameter specification, and prior selection.

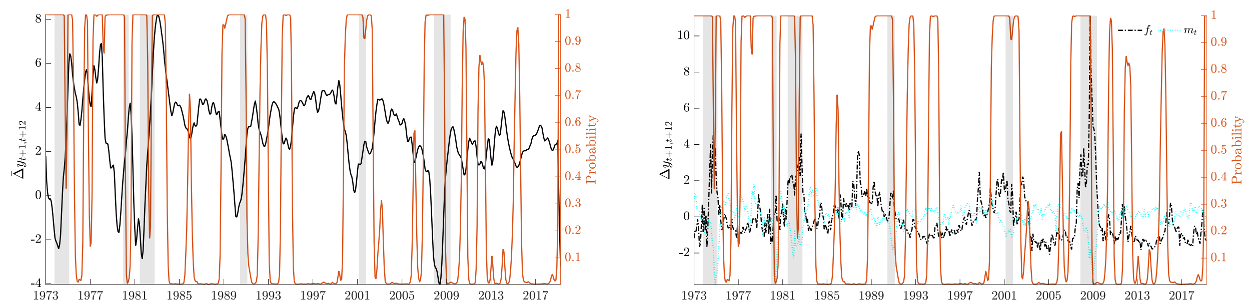
3.3 Results

The financial factor exerts more influence than the macroeconomic factor in the transition from "normal" to "bad" regimes, whereas the macroeconomic factor plays a more critical role when transitioning from "bad" to "normal" regimes. The influence of financial and macroeconomic conditions on these transitions are summarized in Table 1. The point estimates of the coefficients controlling the transition probabilities across regimes have the expected signs, but with substantial uncertainty around them. A tightening of financial conditions leads to an increase in the probability of transitioning from the "normal" to the "bad" regime.

The transitions to and from the "bad" regime are typically associated with very negative and positive realizations of future GDP growth. The left panel in figure 3 shows the estimated probability of observing the "bad" regime, where the transition into the "bad" often precedes and has similar duration as NBER dated recessions. However, there are also periods in which the estimated "bad" regime probability coincides with future GDP growth realizations closer to historical values, and no recorded recession. For example, during the period 1975-1979, there were several instances in which the model suggests a substantial probability in favor of the "bad" regime, yet future GDP growth remained positive. The right panel in figure 3 shows that periods associated with these "false positives" coincide with an increase in financial stress that was not accompanied by a deterioration in macroeconomic conditions.

Figure 3: Estimated "Bad" Regime Probabilities Against Dependent Variable. Transition Probabilities Depend on Financial and Macroeconomic Factor.

Estimated Regime and Future Growth **Estimated Regime and Factors**



NOTE: Estimated "bad" regime probabilities from Markov-switching regression (red), average GDP growth over next year (black), financial factor (dotted-black) and macroeconomic factor (cyan). Regime-switching regression with two regimes and one Markov chain for both coefficients and volatility. Transition probabilities are endogenous and depend both on the financial and the macroeconomic factor.

Table 1: Estimated Endogenous Transition Probabilities.

	p_{12} from Normal to Bad			p_{21} from Bad to Normal	
a_{12}	2.69	[2.32, 3.10]	a_{21}	2.13	[1.75, 2.54]
b_{12}	0.23	[0.09, 0.42]	b_{21}	-0.20	[-0.38,-0.07]
c_{12}	-0.25	[-0.50,-0.06]	c_{21}	0.21	[0.06, 0.48]

NOTE: Values in brackets indicate lower (5%) and upper (95%) bound of coefficients derived from their posterior distribution calculated via MCMC using 20,000 replications, a burn-in of 1,000 replications (thus for a total of 210,000 effective replications). Regime-switching regression with two regimes and one Markov chain for both coefficients and volatility. Transition probabilities are endogenous and depend both on the financial and the macroeconomic factor. Notice that the diagonal elements are not estimated since the state transition probabilities add up to one for a given regime.

Our two-state MS model captures the inverse relationship between average growth and volatility, consistent with the essential feature of Growth-at-Risk. Table 2 reports the estimated coefficients that relate the financial and macroeconomic factors to future GDP growth. The estimates of the parameters $\alpha_y(f_t, s_t)$ and $\sigma_y(f_t, m_t)$ show that average future GDP growth is lower and more volatile in the "bad" regime relative to the "normal" regime. Compared to the "normal" regime, our estimates indicate that average future GDP growth is about one percentage point lower and three times more volatile in the "bad" regime.

Future GDP growth is more sensitive to the macroeconomic and financial factors in the "bad" regime relative to the "normal" regime. These responses are captured by the estimated coefficients $\beta_y(f_t, m_t)$ and $\gamma_y(f_t, m_t)$. As the probability of transitioning to the "bad" regime increases, these increased sensitivities can generate mean and variance co-movements associated with downside risks to the outlook.

Our model also captures an asymmetric effect of macroeconomic and financial conditions on future GDP growth. This feature is present in both regimes with the estimated coefficients $\gamma_y(f_t, m_t) > \beta_y(f_t, m_t)$. While the sensitivity of future GDP growth to the financial factor is about the same in both "normal" and "bad" regimes, it doubles in the "bad" regime for the macroeconomic factor. This asymmetry suggests that macroeconomic conditions can generate additional downside risks when the economy transitions to the "bad" regime.

Table 2: Estimated Regime-Switching Coefficients.
Transition Probabilities Depend on Financial and Macroeconomic Factor.

	Average GDP Growth Over Next Year			
	Bad Regime		**Normal Regime**	
$\alpha_y(f_t, m_t)$	-0.99	[-1.25,-0.75]	0.59	[0.48, 0.68]
$\beta_y(f_t, m_t)$	-0.29	[-0.49,-0.08]	-0.02	[-0.08, 0.04]
$\gamma_y(f_t, m_t)$	0.54	[0.22, 0.91]	0.29	[0.15, 0.41]
$\sigma_y(f_t, m_t)$	2.60	[2.35, 2.89]	0.64	[0.57, 0.71]
	Financial Factor		**Macroeconomic Factor**	
$\alpha_{f/m}$	0.00	[-0.05, 0.04]	0.00	[-0.03, 0.02]
$\beta_{f/m}$	0.88	[0.84, 0.92]	-0.07	[-0.08,-0.05]
γ_f	-0.10	[-0.22, 0.03]		
$\eta_{f/m}$	0.05	[-0.07, 0.17]	0.82	[0.79, 0.86]
$\sigma_{f/m}$	0.63	[0.60, 0.67]	0.34	[0.32, 0.35]

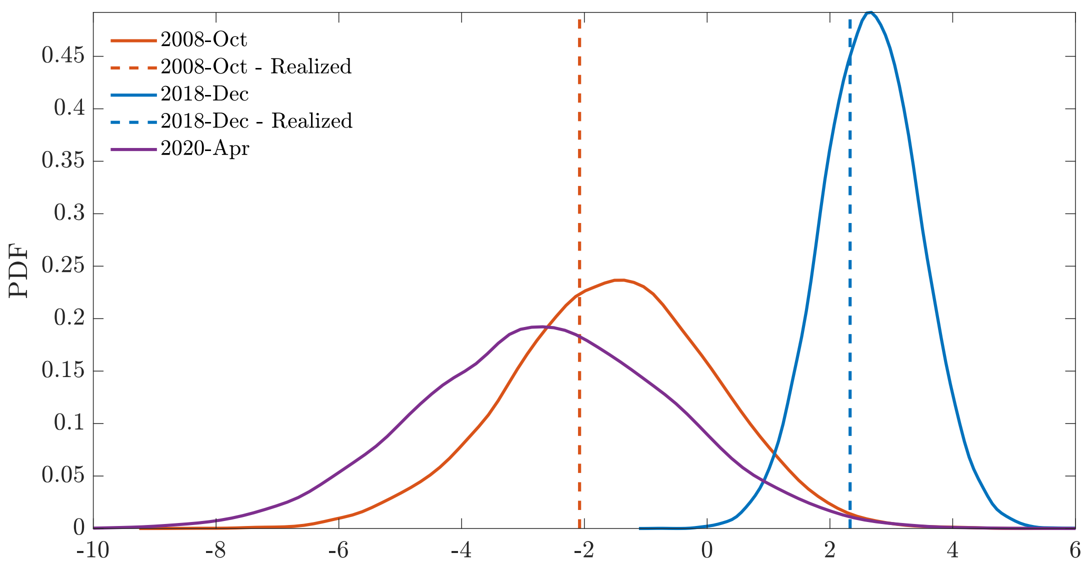
NOTE: Values in brackets indicate lower (5%) and upper (95%) bound of coefficients derived from their posterior distribution calculated via MCMC using 20,000 replications, a burn-in of 1,000 replications (thus for a total of 210,000 effective replications). Regime-switching regression with two regimes and one Markov chain for both coefficients and volatility. Transition probabilities are endogenous and depend both on the financial and the macroeconomic factor.

The conditional distributions of future GDP at specific events bring a clearer image of the build-up of risks to the outlook. We use the two-state MS model estimates and pick periods representing the conditions associated with the "normal" regime and two periods associated with the "bad" regime, but with different driving forces in terms of the macroeconomic and financial factors. For the "normal" regime, we take as a starting point December-2018, and forecast GDP growth from January to December 2019, period characterized by calm conditions, as expressed by low financial and macroeconomic factors. For the "bad" regime, we start in October-2008, and consider the distribution of GDP growth from November-2008 to October-2009, to capture the peak of the Great Financial Crisis and the associated financial stress as the primary source of risks to the outlook. As a third period, we pick April-2020, in which the "bad" regime corresponds to the sharp decline in the macroeconomic factor due to the disruption in economic activity associated with lock-downs and work-from-home orders put in place in response to the spread of the COVID-19 virus in the U.S.

The conditional distributions of future GDP clearly show a shift in mean and variance when the economy transitions from "normal" to "bad" regimes. Figure 4 shows slices of the conditional distribution of future GDP growth corresponding to the three events previously described. In December-2018, consistent with our estimate of the U.S. economy being in the "normal" regime, the conditional distribution of future GDP growth is tightly centered around 3%, with most of the mass of expected growth between 1% and 4%. The blue dashed line shows the realized value of GDP growth, which turned out to be reasonably close to the conditional predictive distribution mode. The distribution constructed in November-2008 illustrates the opposite situation. With the U.S. economy estimated to be in the "bad" regime, the distribution of future GDP growth is shifted to the left and centered around -1% with an apparent increase in the variance of expected outcomes, and in particular of a substantial increase in the probability of observing negative growth rates. In this case, realized GDP growth turned out in the vicinity of the mode of the predictive distribution.

Lastly, looking at the predictive distribution on April-2020, and assuming that the U.S. has already transitioned into the "bad" regime, we see a similar shift in the mean and variance of the distribution of future GDP growth. Even though financial conditions had already stabilized in April, the sheer size of the economic activity decline pushed the distribution further into the negative growth territory.

Figure 4: Predictive Densities: Selected Episodes



NOTE: Densities of average GDP growth over the next year from Markov-switching regression in example episodes of the bad regime (red) and the normal regime (blue). The model features one Markov chain for both coefficients and volatility. Transition probabilities are endogenous and depend both on the financial and the macroeconomic factor. The figure also shows the distribution in April 2020, at the peak of the Covid-19 pandemic. Since the last available observation for average future GDP growth in our sample is May 2019, we assume that this period would fall into a bad regime and use the coefficients associated with that regime to generate the purple distribution.

3.4 Agnostic Transition Probabilities

Before we close this section, we take a step back and estimate a version of the model where we are agnostic about the relationship between financial and macroeconomic factors and the transition probabilities of the two regimes. We do so to isolate the effect of the time-varying elasticity of future GDP growth from the time-varying transition probabilities we investigated previously. In particular, we consider the following specification of the regime-switching model:

$$\bar{\Delta} y_{t+1,t+12} = \alpha_y(s_t) + \beta_y(s_t) f_t + \gamma_y(s_t) m_t + \sigma_y(s_t)\varepsilon_t^y, \quad (8)$$

$$f_t = \alpha_f + \beta_f f_{t-1} + \gamma_f m_t + \eta_f m_{t-1} + \sigma_f \varepsilon_t^f, \quad (9)$$

$$m_t = \alpha_m + \beta_m f_{t-1} + \eta_m m_{t-1} + \sigma_m \varepsilon_t^m, \quad (10)$$

where the coefficients $\theta(s_t) \equiv \{\alpha(s_t), \beta(s_t), \gamma(s_t)\}$ and the standard deviation $\sigma(s_t)$ vary depending on an unobserved regime variable $s_t \in \{1,2\}$, which indicates the regime prevailing at time t. The latent variable s_t is governed by a discrete time, discrete state Markov stochastic process, defined by the transition probabilities:

$$p_{ij} \equiv Pr\left(s_{t+1} = j | s_t = i\right), \quad \sum_{j=1}^{2} p_{ij} = 1, \quad \forall i,j \in \{1,2\}. \quad (11)$$

We estimate this model using the same sample and methods used for the model with endogenous transition probabilities. Detailed estimation results are provided in Appendix B.[6]

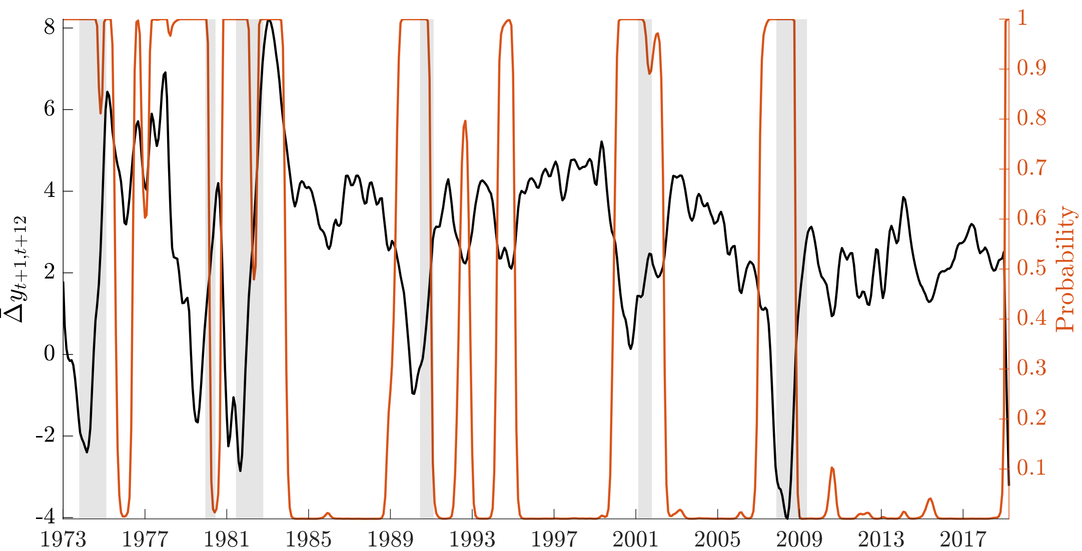
Under this alternative model, the elasticities of future GDP growth to the financial and macroeconomic factors asymmetrically increase in the "bad" regime, and macroeconomic conditions have a more substantial impact than financial conditions. The "normal" regime has an average duration of about three years, roughly twice as long as the average duration of the "bad" regime. The probability of transitioning from the "normal" to the "bad" regime is lower than transitioning from "bad" to "normal," which implies that the "bad" regime is much less frequent in the data. Compared to the "normal" regime, the average future GDP growth in the "bad" regime is about 1.5 percentage points lower, and its volatility is four times larger. The parameters governing the evolution of the financial and macroeconomic

[6]Appendix B also shows that estimating the model without equations (9) and (10) gives very similar results.

factors are similar to those in the endogenous transition model.

Like in the endogenous transition model, the "bad" regime occurs around periods associated with U.S. recessions and extreme realizations of expected future GDP growth. Figure 5 shows the time path of the estimated probability corresponding to the U.S. economy in the "bad" regime. The model also associates the "bad" regime with periods of higher than usual financial stress or below average macroeconomic conditions without large movements of expected future GDP growth. Compared to Figure 3, the transition to the "bad" regime is sharper and without occasional spikes, given that we abstract from the indirect effect of the macroeconomic and financial factors on the regime transition probabilities. However, our message's essence remains, with growth-at-risk associated with a transition to a regime with higher volatility and lower expected growth.

Figure 5: Estimated "Bad" Regime Probabilities.



NOTE: Estimated "bad" regime probabilities from Markov-switching regression (red) against average GDP growth over next year (black). Regime-switching regression with two regimes and one Markov chain for both coefficients and volatility.

4 A Unifying Framework for Tail Risks: Regime Switching and Quantile Regressions

Markov-Switching Models (MS) and Quantile Regressions (QR) are two alternative approaches for quantifying tail risks. While the first is a more parametric approach with a clear structure linking shifts in the mean and variance, the second is a more flexible system that lets the data drive the results. We show that MS and QR models agree upon quantifying tail risks and their determinants, producing similar results. To that end, we allow the MS model to have a full-blown set of ingredients, sufficiently capturing any non-linearity emerging from the macroeconomic or financial side. We compare this fully-flexible MS process with a monthly version of the QR procedure proposed by Adrian *et al.* (2019). In particular, we show how key features of QR distributions are consistent with co-movement in mean and variance across regimes in a model where both macro and financial conditions matter. As such, the MS model offers one possible parametric interpretation of the QR results.

Our first model is the Markov-Switching process with three regimes ("bad," "normal" and "good"), with two separate Markov chains, s_t and c_t, governed by endogenous probabilities depending on the macroeconomic and financial factor, one governing the volatilities and the other the constant and coefficients, decribed by:

$$\bar{\Delta} y_{t+1,t+12} = \alpha_y(c_t) + \beta_y(c_t) f_t + \gamma_y(c_t) m_t + \sigma_y(s_t)\varepsilon_t^y, \tag{12}$$

$$f_t = \alpha_f + \beta_f f_{t-1} + \gamma_f m_t + \eta_f m_{t-1} + \sigma_f \varepsilon_t^f, \tag{13}$$

$$m_t = \alpha_m + \beta_m f_{t-1} + \eta_m m_{t-1} + \sigma_m \varepsilon_t^m, \tag{14}$$

where the coefficients $\theta_y(c_t) \equiv \{\alpha_y(c_t), \beta_y(c_t), \gamma_y(c_t)\}$ and the standard deviation $\sigma_y(s_t)$ vary depending on unobserved regime variables $c_t \in \{1,2,3\}$ and $s_t \in \{1,2,3\}$ which indicate the state prevailing at time t for a total of 9 possible regimes (combinations of states c_t and s_t).

The transition probabilities governing average future growth and the elasticity to the financial and macroeconomic factors $p_{ij} \equiv Pr\left(c_{t+1} = j | c_t = i\right), \quad \sum_{j=1}^{3} p_{ij} = 1, \quad \forall i, j \in \{1, 2, 3\}$, and the transition probabilities governing the variance of future growth, $q_{kl} \equiv Pr\left(s_{t+1} = j | s_t = i\right)$, $\sum_{k=1}^{3} q_{kl} = 1, \quad \forall k, l \in \{1, 2, 3\}$, depend on the financial factor f_t and the macroeconomic factor m_t through logistic functions exactly as in the two-state model described in section 3. Estimated parameters of the endogenous transition probabilities and estimated regime probabilities are provided in Appendix B.

Our second model is a Quantile Regression (QR), which is a flexible tool for studying the determinants of tail risks to growth, or Growth-at-Risk.[7] Our growth measure of interest is the (annualized) average growth rate of GDP one year ahead, or between month $t+1$ and $t+12$, $\bar{\Delta} y_{t+1,t+12}$. We consider a linear model for the conditional GDP growth quantiles whose predicted value

$$\widehat{\mathcal{Q}}_\tau(\bar{\Delta} y_{t+1,t+12} | x_t) = x_t \hat{\beta}_\tau, \tag{15}$$

is a consistent linear estimator of the quantile function of $\bar{\Delta} y_{t+1,t+12}$ conditional on x_t – where $\tau \in (0,1)$, x_t is a $1 \times k$-dimensional vector of conditioning (risk) variables, and $\hat{\beta}_\tau$ is a $k \times 1$-dimensional vector of estimated quantile-specific parameters.[8] Accordingly, a determinant x_t is non-linearly related to GDP growth if it affects the median and the tails differently.

Our estimation framework follows Adrian *et al.* (2019) and López-Salido and Loria (2020), with the important difference of moving to monthly frequency, allowing our model for a timely assessment of developments in financial markets and the real economy. To that end,

[7] For an introduction to the quantile regression methodology, see Koenker (2005).

[8] The relationship between x_t and a quantile $\tau \in (0,1)$ of $\bar{\Delta} y_{t+1,t+12}$ is measured by the coefficient $\hat{\beta}_\tau$:

$$\hat{\beta}_\tau = \underset{\beta_\tau \in \mathbb{R}^k}{\operatorname{argmin}} \sum_{t=1}^{T-h} \left(\tau \cdot \mathbb{1}_{(\bar{\Delta} y_{t+1,t+12} \geq x_t \beta)} |\bar{\Delta} y_{t+1,t+12} - x_t \beta_\tau| + (1-\tau) \cdot \mathbb{1}_{(\bar{\Delta} y_{t+1,t+12} < x_t \beta)} |\bar{\Delta} y_{t+1,t+12} - x_t \beta_\tau| \right),$$

where $\mathbb{1}_{(\cdot)}$ denotes the indicator function, taking the value one if the condition is satisfied.

we consider our real-time macroeconomic (m_t) and financial factors (f_t) described in Section 2 as conditioning variables in equation (15):

$$\widehat{\mathcal{Q}}_\tau(\bar{\Delta}y_{t+1,t+12}|x_t) \quad = \quad \hat{\alpha}_\tau + \hat{\beta}_\tau f_t + \hat{\gamma}_\tau m_t, \tag{16}$$

where $\Delta y_{t+1,t+12}$ is calculated from our monthly GDP series.

The estimated conditional quantiles are approximations to the so-called "quantile function", that is, $\mathcal{Q}_\tau(\bar{\Delta}y_{t+1,t+12}|x_t) = F^{-1}(\bar{\Delta}y_{t+1,t+12}|x_t)$, where $F^{-1}(\cdot)$ is the conditional inverse cumulative distribution function (CDF) of average future GDP growth. As noted by Adrian *et al.* (2019), it is challenging to map these estimates into a probability distribution function (PDF) because of approximation error and estimation noise. We follow their approach by smoothing the quantile function using the skewed $t-$distribution proposed by Azzalini and Capitanio (2003). Appendix C provides details on the smoothing procedure, the slope coefficients[9] on the financial and the macroeconomic factor, selected conditional distributions and conditional quantiles of average GDP growth over the next year.

As we have argued, a defining feature of growth-at-risk is the inverse relation between mean and volatility. We now show that the MS and QR models provide strikingly similar characterizations of these features. Moreover, both approaches have quantitative and qualitative similar implications in terms of decomposing historical U.S. GDP growth data. Figure 6 presents the fitted values of the MS and the QR models.

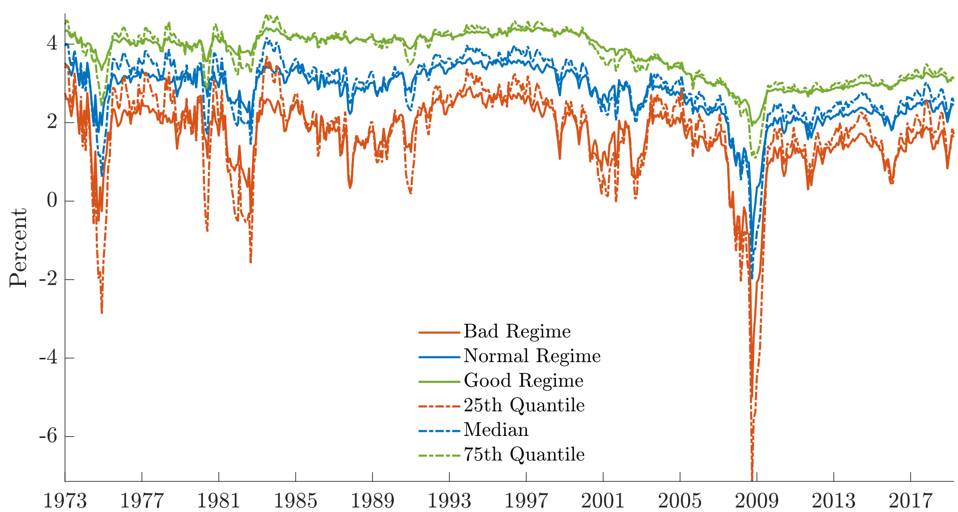
The historical predictive distributions of one-year-ahead GDP growth highlights the similarities between the models. We report regime-specific fitted values for the MS model, while we report the 25$^{\text{th}}$, 50$^{\text{th}}$ and 75$^{\text{th}}$ quantiles for the QR. Figure 6 shows that the "bad" regime from the MS model tracks the 25$^{\text{th}}$ quantile of QR model, the "Good" regime in the MS

[9] We performed ANOVA tests of equality of these slope coefficients across quantiles for a given variable. The test rejects the null hypothesis of equality of the financial factor slopes between the 10$^{\text{th}}$ and the 50$^{\text{th}}$ quantile at a 0.1% significance level. Similarly, we reject the null at a 0% significance level for the macroeconomic factor. The test cannot reject equality of coefficients between the 50$^{\text{th}}$ and the 90$^{\text{th}}$ quantile.

model tracks the 50th quantile, and the "normal" regime aligns with the 75th of the predictive growth distribution.

Both models capture downside and upside risks in future GDP growth. In the MS model, the lower GDP growth and higher variance periods correspond to the "bad" regime. In the QR model, the 25th quantile features lower future growth and is more volatile than the other quantiles. On the flip side, the "Good" regime in the MS model captures a period of higher than average growth and lower volatility, consistent with the 75th quantile in the QR model, and in line with the simple historical inspection of the evolution of monthly GDP (Figure A-2 in the Appendix).

Figure 6: Predictive Growth Distributions: Markov-Switching vs. Quantile Regression.



NOTE: The figure reports regime-specific fitted values of average GDP growth over the next year along with the conditional quantiles of that same variable estimated from the quantile regression. The model features separate, independent Markov chains governing both coefficients and volatility. Transition probabilities are endogenous and depend both on the financial and the macroeconomic factor.

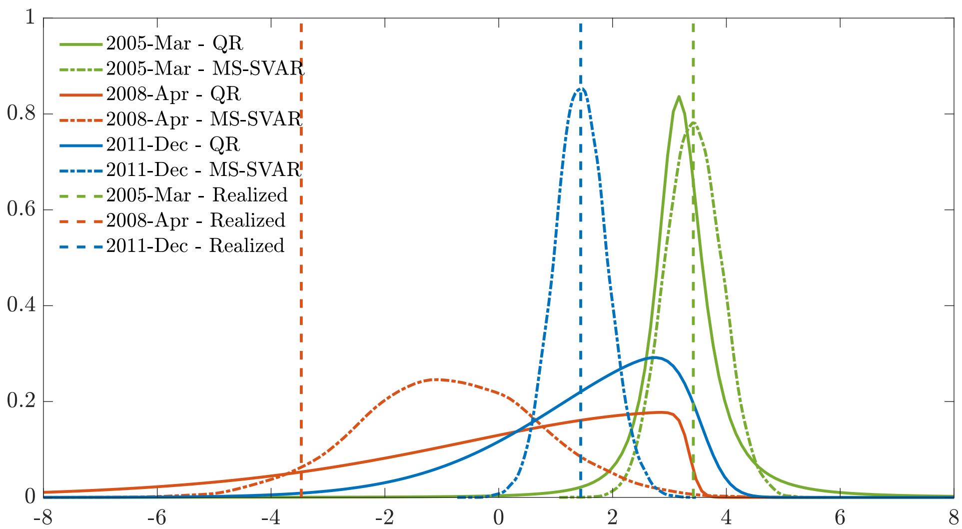
Specific events bring additional insights about how the MS and QR models capture the build-up of risks to the outlook. Figure 7 presents the predictive densities for three different episodes. We focus on a normal period (March 2005), a period with severe financial stress (April 2008) associated with the Great Financial Crisis, and a period of moderate financial stress (December 2011) associated with the debt-ceiling crisis. In April 2008, the predictive distribution of future GDP growth in the MS and QR models was essentially identical and centered around 3%. Consistent with this result, the MS model identifies this period with the "normal" regime, while in the QR model, the fitted distribution is roughly symmetrical, suggesting the absence of downside risks.

As expected, the periods of the severe financial stress associated with the Great Financial Crisis feature a shift towards lower average future GDP growth and an increase in the variance of outcomes. Both the MS and QR models interpret this historical episode as one with substantial downward risks, with over 70% probability of observing GDP growth in negative territory. Lastly, the period of December-2011 shows some of the differences between the MS and QR models. In this episode, the MS model captures the decline in future GDP growth but does not feature an increase in the probability of deficient future outcomes. In contrast, the QR model identifies this period as one in which growth had turn vulnerable, with a substantial increase in downside risks.

The estimated coefficients confirm the similarity of these two approaches to the relation between tail risks and the financial and macroeconomic conditions. Table 3 presents the estimated coefficients for the MS and the QR models. The first insight from the coefficients is that, on average, future GDP growth is lower in the "bad" regime of the MS model and the 25^{th} quantile of the QR model, than in the "Good" regime or the 75^{th} quantile. Interestingly, the MS constants $\alpha_y(c_t)$ and QR constants α_τ are almost identical: GDP growth is, on average, around -1% on the "bad" regime / 25^{th} quantile, and around 1% on the "good" regime / 75^{th} quantile.

The coefficients for the QR clearly show the asymmetric effects of both macroeconomic and financial factors. The coefficients for the financial factor (β_τ) are negative and significant for all quantiles, and positive and significant for the macroeconomic factor (γ_τ), confirming that both macroeconomic and financial conditions matter for the tail risks to the outlook.

Figure 7: Densities: Regime-Switching vs. Quantile Regression.



NOTE: Densities of average GDP growth over the next year from Markov-switching regression and from skewed-t approximation of estimated conditional quantiles from quantile regression. The model features separate, independent Markov chains governing both coefficients and volatility. Transition probabilities are endogenous and depend both on the financial and the macroeconomic factor.

However, the lower tail reacts more to financial distresses and economic downturns than the upper tail. Compared to the 75th quantile, the coefficients for the 25th quantile are about seven times higher for the financial factor and double for the macroeconomic factor. Financial and macroeconomic deterioration markedly increase downside risks to the outlook, while the effect on upside risks is more muted. This asymmetry implies that periods of economic downturn indicate a lower average future GDP growth and higher variance of future outcomes, but with risks tilted to the downside.

The flexibility of the QR model makes it straightforward to assert the inverse relation between mean and volatility by relying on the direct elasticities of macroeconomic and financial conditions to the quantiles. For the MS model, however, this assessment is more complex due to the nine combinations between "bad," "normal," and "good" regimes and low, moderate, and high volatilities. "normal" and "good" regimes are periods of low or moderate variance. The "bad" regime, in turn, has more events of the high variance than the other two regimes combined. In other words, periods of high variance are predominantly marked by low growth, in line with the QR evidence. Figure B-5 in the Appendix helps understanding this relationship.

The results of both models presented here bring empirical evidence to a unified framework of tail risks to the outlook. In the next section, we evaluate how to make use of such tools for risk assessment and policy responses.

Table 3: Estimated Coefficients of Markov-Switching and Quantile Regression Models

	Regime-Switching					
	Three Regimes and Separate Markov Chains for Coefficients and Volatility					
	Bad Regime		***Normal Regime***		***Good Regime***	
$\alpha_y(c_t)$	-1.03	[-1.13,-0.92]	0.06	[-0.01, 0.14]	1.03	[0.96, 1.09]
$\beta_y(c_t)$	-0.55	[-0.66,-0.44]	-0.30	[-0.39,-0.19]	-0.05	[-0.10, 0.00]
$\gamma_y(c_t)$	0.08	[-0.16, 0.39]	0.01	[-0.14, 0.18]	0.15	[0.01, 0.30]
	Low Volatility		**Mild Volatility**		**High Volatility**	
$\sigma_y(s_t)$	0.24	[0.21, 0.27]	0.45	[0.38, 0.54]	2.89	[2.55, 3.31]
	Quantile Regression					
	25th Quantile		***Median***		***75th Quantile***	
α_τ	-0.99	[-1.11,-0.86]	0.20	[0.11, 0.29]	1.02	[0.96, 1.08]
β_τ	-0.60	[-0.71,-0.48]	-0.29	[-0.37,-0.21]	-0.09	[-0.14,-0.04]
γ_τ	0.68	[0.42, 0.94]	0.38	[0.21, 0.55]	0.33	[0.20, 0.46]

NOTE: Values in brackets for regime-switching models indicate lower (5%) and upper (95%) bound of coefficients derived from their posterior distribution calculated via MCMC using 20,000 replications, a burn-in of 1,000 replications (thus for a total of 210,000 effective replications). Values in brackets for regime-switching models indicate lower (16%) and upper (84%) bound of coefficients computed via "blocks-of-blocks" bootstrap (see Appendix C) using 500 replications.

5 Risk Assessment

We have shown that tail risk models that link financial and macroeconomic conditions to future growth capture the essential features of growth-at-risk. We agree that these models approximate the underlying distribution of expected growth and its determinants and might be imperfect tools for predicting the exact magnitude of a crisis. However, forecasting accuracy in mean or modal outcomes is not the only component of risk assessment. Risk management is central to policy-making institutions that base decisions on inputs coming from multiple sources, including formal models and judgemental projections. In this context, we show that growth-at-risk models provide valuable input for decision-makers to quantify risks and uncertainties to the outlook.

When risks of adverse outcomes are elevated relative to historical experience, risk management considerations might require policy actions even if the forecast is inaccurate or the risks do not materialize. We illustrate the use of growth-at-risk models in the context of risk management by examining the early stages of the global financial crisis and build-up of risk at the onset of the COVID-19 pandemic in the U.S.

5.1 Risk to the Outlook During the Global Financial Crisis

Risk management considerations were crucial in driving policy easing by the FOMC during the initial stages of the global financial crisis, with downside risks coming from financial conditions. In the second half of 2007 and the first half of 2008, the FOMC described in its statements an outlook of moderate growth subject to substantial downside risks coming from tight financial conditions.[10] For instance, on August 16, 2007, the FOMC stated that

[10] The Survey of Professional forecasters provides a similar assessment. For instance, in the February 2008 release of the SPF, consensus GDP growth between 2008:Q2 and 2009:Q1 was 2.5 percent (against 1.5 from the Federal Reserve Board's staff and a realized -4.9 percent). However, the release flagged the high risk of a contraction (for instance, 43 percent probability of negative growth in 2008 Q2, up from 22 percent for the

"*Financial market conditions have deteriorated, and tighter credit conditions and increased uncertainty have the potential to restrain economic growth going forward. In these circumstances, although recent data suggest that the economy has continued to expand at a moderate pace, the Federal Open Market Committee judges that the downside risks to growth have increased appreciably.*" Similar language was adopted in September 2007, and January and March 2008.[11]

We evaluate whether the Quantile Regression model provides useful input to quantify risks and uncertainty around the outlook during a period of financial stress. Specifically, we construct the distribution of average U.S. GDP growth over the next year for the four months corresponding to the FOMC statements mentioned above. We run a pseudo-out-of-sample forecast by stopping the estimation in the month of the FOMC meeting. Figure 8 plots the resulting Quantile Regression conditional distributions in blue. For each month considered, the green vertical line denotes the Federal Reserve Board's staff forecast for average GDP growth over the next year in that month's Teal book, whereas the red vertical line denotes the ex-post data realization.

In terms of point forecasts, the Fed staff's forecasts and the Quantile Regression model agree. The median forecast of the distributions from the Quantile Regression model is close to the Federal Reserve Board's staff forecast, which we take to represent the FOMC's interpretation of the most likely outcome for the average GDP growth in each of these episodes. However, the actual realization was considerably lower than the staff's forecasts and the median forecast from both models. A policymaker focusing only at point estimates would miss the significant downside risks imposed by the financial and macroeconomic deterioration of the global financial crisis.

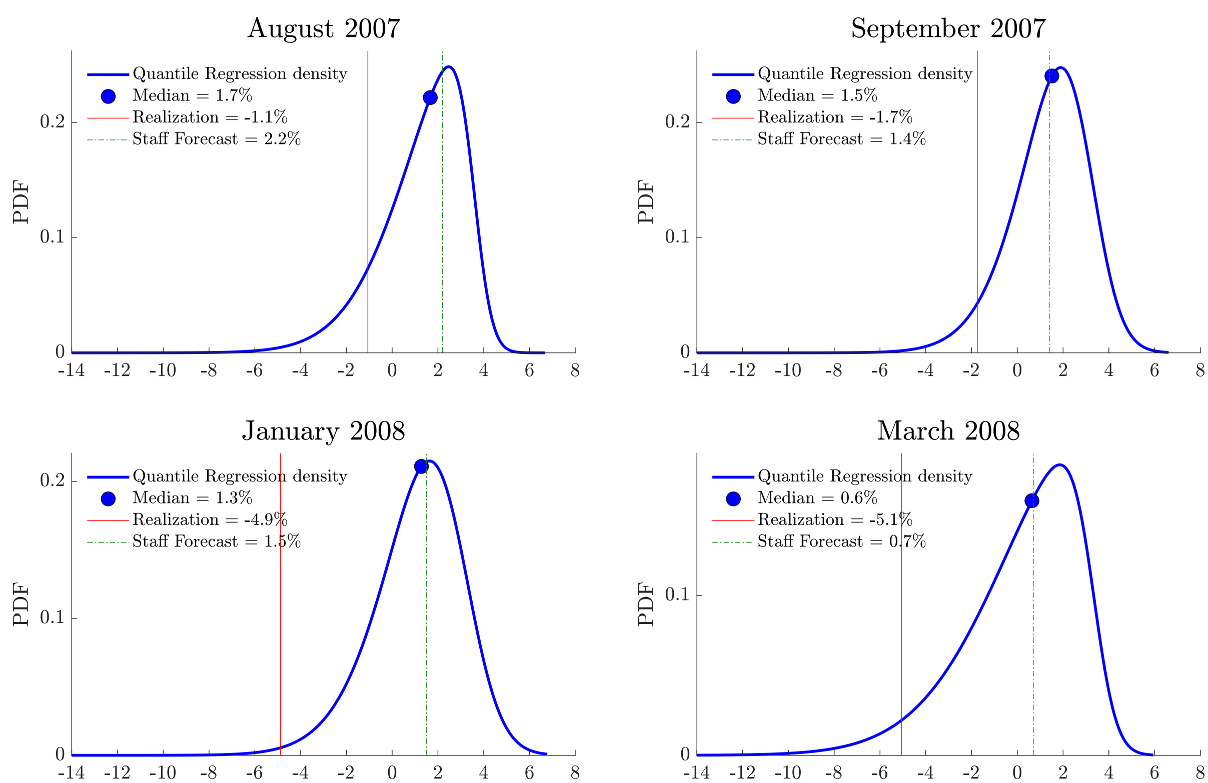
The Quantile Regression model would have predicted such a sharp economic downturn,

previous release. Consensus for 2008:Q2 was 1.3.

[11]The statements are reported in Appendix E.

way beyond the policymakers' expectations. As discussed before, this model indicates significant downside risks to the outlook in the light of financial and macroeconomic deterioration. It turns out that the model assigned a non-negligible probability of what eventually became the GDP growth realization, even though the median forecast were substantially higher. A policymaker with such a tool in hand at that time would have had a much richer assessment of the potential economic downturn ahead.

Figure 8: Out-of-Sample Conditional Distributions of Average U.S. GDP Growth over the Next Year.



NOTE: For a given period, the density is computed by fitting a skewed-t distribution on the conditional quantiles for that same period. In particular, we estimate a quantile regression starting in January 1973 and stopping one year before the selected period (e.g., August 2006 for the August 2007 density since that this is the last available date for average GDP growth over the next year). We then use the coefficient estimates and the data on the financial and the macroeconomic factor to construct the quantiles for the selected period (e.g., August 2007 for average growth between September 2007 and August 2008).

5.2 Tracking COVID-19 Build-Up of Risks

We now use our framework to assess the build-up of risk during the health crisis triggered by the COVID-19 pandemic. Restrictions on mobility and social interaction in the U.S. rapidly disrupted economic activity. We take advantage of our framework's real-time nature and its ability to incorporate weekly labor market variables that provided the first signals about the extent of the economic collapse to evaluate the potential risks for the year ahead.

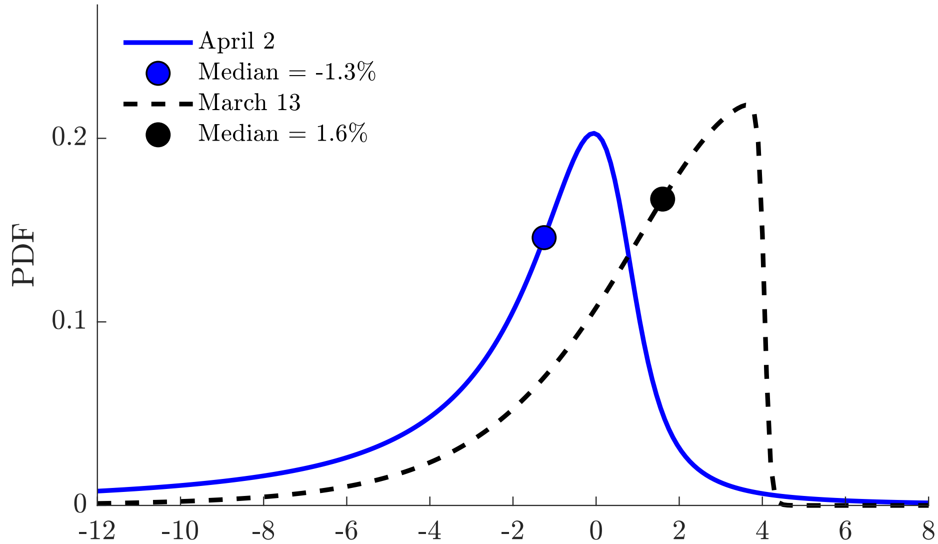
The Quantile Regression model indicates a sharp expected economic downturn, and the downside risks are unprecedented. Figure 9 plots the resulting real-time conditional distributions from the Quantile Regression model evaluated in March 13, 2020 (dashed black) and in April 2, 2020 (blue). The rapid deterioration of the macroeconomic and financial conditions brought severe uncertainty to the outlook, with substantially high probabilities of observing negative growth. While risks were already tilted to the downside in March, with a distribution substantially skewed to the left, the median moved from a positive outcome (1.6%, in March) to negative (-1.3%, in April) in just one month. Although many unknown factors do not directly map into our framework, such as the effect of medical developments, the potential of a second wave of the virus, or supply chain disruptions, tail risk models paint an informative picture of what to expect for the months ahead.

6 Robustness

We examined several alternatives to verify the robustness of our results, which can be classified in two broad groups: conditioning variables and model specification. We briefly summarize the results.

In terms of conditioning variables, we explored alternative financial and macroeconomic indicators. Our results are robust to using the Chicago Fed National Financial Conditions

Figure 9: Conditional Distribution of U.S. Outlook in March 2020 Across Vintages.



NOTE: Conditional distributions of average GDP growth over the next year in March 2020, obtained by fitting skewed-t distribution on conditional quantiles obtained from quantile regression across two vintages. The March 13$^{\text{th}}$ vintage does not include the initial unemployment insurance claims from the second half of March, whereas the April 2$^{\text{nd}}$ vintage does.

Index and its sub components (Brave and Kelly, 2017). Similarly, we have explored results using the U.S. component of the European Central Bank Composite Indicator of Systematic Stress in the Financial System (Kremer *et al.*, 2012) and The Kansas City Financial Stress Index (Hakkio and Keeton, 2009). Our results are also robust to replacing our macroeconomic factor with the Philadelphia Fed Aruoba-Diebold-Scotti Business Conditions Index (Aruoba *et al.*, 2009) or using GDP growth at a quarterly frequency as in (Adrian *et al.*, 2019). Appendix D shows details of these exercises as well as additional experiments in which we experimented with different sample splits.

As additional robustness checks, we tested alternative specifications of the MS model. In particular we estimated the three regime with a single Markov Chain governing the elasticity to the financial and macro factor as well as the volatility of future GDP growth. The advantage of this model is that it is somewhat easier to interpret as periods of low expected growth coincide with periods of higher variance.

7 Conclusion

We provide a framework to understand the unique features of persistent historical swings in aggregate economic activity and its volatility. We show that economic activity in the U.S presents (at least) two well-characterized regimes: a high growth/low variance associated with expansions, such as the U.S. Great Moderation during the 1990s and part of the 2000s, and a low growth/high variance, such as the Great Recession of 2008-2009. By quantifying the connection between economic conditions and the time variation in the mean and volatility of expected future growth using a Markov-Switching model, we show that both financial and macroeconomic conditions matter to explain these features. Moreover, the regimes mapped by the Markov-Switching structure constitutes the foundation behind growth-at-risk models. Quantifying risks and uncertainties to the outlook in real-time, and understanding its sources, allow decision-makers to have a clearer picture of the road ahead, making course adjustments an easier and more efficient task.

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A Data Appendix

A.1 Data sources

Table A-1: Description of Macroeconomic and Financial Variables.

	Name	Description	Source
1	Industrial Production	Annualized monthly growth rate of industrial production.	FRED
2	Retail Sales	Annualized monthly growth rate of retail sales.	FRED
3	PMI-NEO	New Export Order component of the Purchasing Managers' diffusion Index.	Haver Analytics
4	Initial Claims	Initial claims for unemployment Insurance as share of civilian labor force.	Haver Analytics
5	GDP	Annualized quarterly growth rate of Gross Domestic Product (GDP).	FRED
6	VXO	CBOE S&P 100 VXO volatility index.	FRED
7	EBP	Excess bond premium as computed by Gilchrist and Zakrajšek (2012).	Favara *et al.* (2016)'s website
8	TED Spread	Difference between the 3-month LIBOR rate and the 3-month Treasury bill.	Haver Analytics
9	CBILL Spread	difference between the 3-month financial commercial paper rate and the 3-month Treasury bill.	Haver Analytics

NOTE: Industrial Production, Retail Sales, Initial Claims, EBP and CBILL Spread available from Jan/1973, GDP available from 1973:Q1, TED Spread available from Jun/1980, VXO available from Jan/1986 and PMI-NEO available from July/2007.

A.2 Illustration of Information Flow

Take the first month of a quarter ($M1$), and call it January. The available information about $M1$ is small, with initial unemployment claims as the only data released. In February, we receive additional information about January with PMI data released early in the month, and industrial production and retail sales in the second half of February. Also, we typically observe an initial release of GDP corresponding to the previous quarter. Finally, in March, we typically have all the January indicators and some information for February, while the only available information for March is initial claims.

Table A-2: Example of Information Flow in the United States.

January

		Industrial Production	Retail Sales	PMIs	Initial Claims	GDP
Q4	M12	Y	Y	Y	Y	N
Q1	M1	N	N	N	Y	N

February

		Industrial Production	Retail Sales	PMIs	Initial Claims	GDP
Q4	M12	Y	Y	Y	Y	Q
Q1	M1	Y	Y	Y	Y	N
Q1	M2	N	N	N	Y	N

March

		Industrial Production	Retail Sales	PMIs	Initial Claims	GDP
Q4	M12	Y	Y	Y	Y	Q
Q1	M1	Y	Y	Y	Y	N
Q1	M2	Y	Y	Y	Y	N
Q1	M3	N	N	N	Y	N

NOTE: N indicates that the observation is missing. Y denotes monthly observation available for majority of countries in our sample. Q denotes that quarterly average are available. Dashed lines separate the available data (Y and Q) from missing (N), denoting the "ragged-edge" pattern.

A.3 Additional Results Financial and Macroeconomic Factors

Table A-3: Correlations Between Macroeconomic Factor and Underlying Data.

	Macro Factor	Industrial Production	Retail Sales	PMI-NEO	Initial Claims
Macro Factor	1.00	-	-	-	-
Ind. Production	0.78	1.00	-	-	-
Retail Sales	0.28	0.27	1.00	-	-
PMI-NEO	0.74	0.52	0.23	1.00	-
Initial Claims	-0.74	-0.42	-0.10	-0.56	1.00
GDP	0.77	0.47	0.29	0.66	0.46

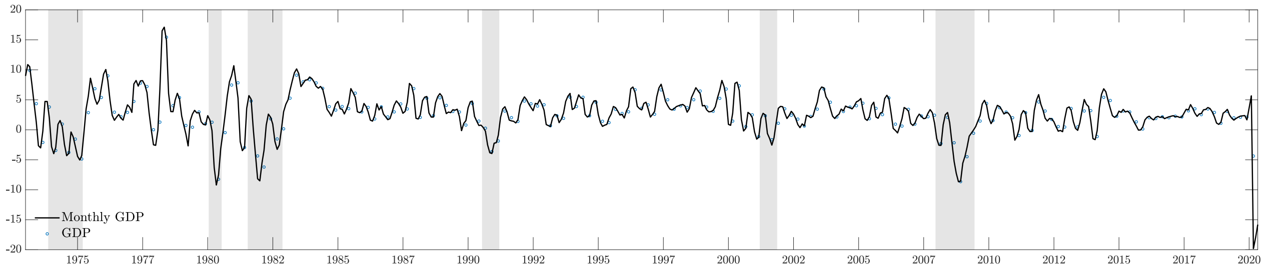
Note: Macroeconomic factor constructed using a Dynamic Factor Model as described in Section 2. Underlying data is the annualized monthly growth rate of industrial production (IP) and retail sales (RS), the New Export Order component of the Purchasing Managers' diffusion Index (PMI-NEO), initial claims for unemployment Insurance as share of civilian labor force (Initial Claims) and the annualized quarterly growth rate of Gross Domestic Product (GDP). PMI-NEO is available from July/2007 and all other series from Jan/1973. For comparison, all data series are detrended and expressed in standardized units. Shaded areas denote NBER dated recession months.

Table A-4: Correlations Between Financial Factor and Underlying Data.

	Financial Factor	VXO	EBP	TED Spread	CBILL Spread
Financial Factor	1.00	-	-	-	-
VXO	0.83	1.00	-	-	-
EBP	0.84	0.66	1.00	-	-
TED Spread	0.80	0.51	0.43	1.00	-
CBILL Spread	0.79	0.50	0.39	0.92	1.00

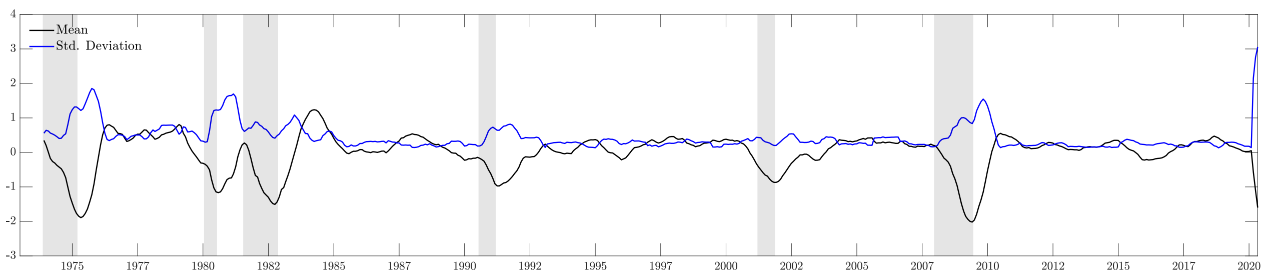
Note: Financial factor constructed using a Dynamic Factor Model as described in Section 2. Underlying data are the CBOE S&P 100 VXO volatility index (VXO), the excess bond premium (EBP), the TED spread (the difference between the 3-month LIBOR rate and the 3-month Treasury bill) and the CBILL spread (the difference between the 3-month financial commercial paper rate and the 3-month Treasury bill). EBP and CBILL spread are available from Jan/1973, TED spread is available from Jun/1980 and the VXO is available from Jan/1986. Shaded areas denote NBER dated recession months.

Figure A-1: Monthly GDP.

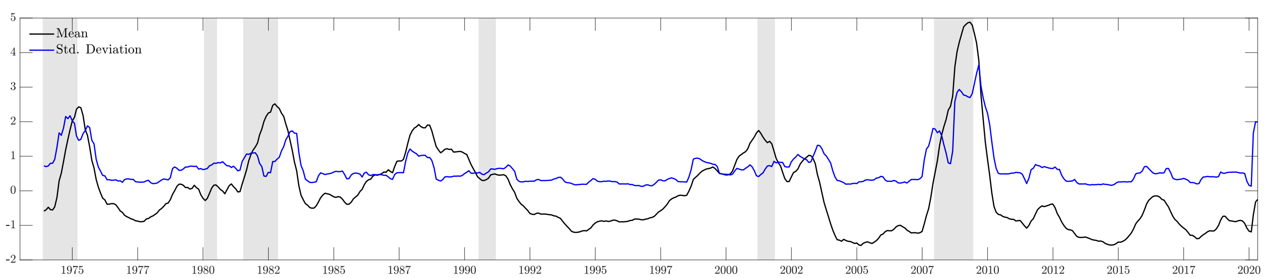


NOTE: Monthly GDP constructed using a Dynamic Factor Model as described in Section 2.1, and underlying data as described in Section 2.2. Shaded areas denote NBER dated recession months.

Figure A-2: 12-month Rolling Window of Mean and Standard Deviation - Macroeconomic and Financial Factors and Monthly GDP.



(a) Macroeconomic Factor.



(b) Financial Factor.

NOTE: 12-month rolling window of mean and standard deviations of Macroeconomic and Financial factors constructed using a Dynamic Factor Model as described in Section 2.1. Shaded areas denote NBER dated recession months.

B Details on Markov-Switching Regression

B.1 Estimation Procedure and Priors

We solve the model in the RISE toolbox[12] using the perturbation methods developed by Maih (2015). The model is then estimated using Bayesian methods with prior hyperparameters specified in Table B-1. We choose a Minnesota-type prior used in Dynare (see Villemot and Pfeifer, 2017) for the coefficients and a Dirichlet prior for the transition probabilities. We estimate the model using detrended monthly GDP growth and add the trend back when computing fitted values.

Table B-1: Prior Hyperparameters.

Coefficients: Minnesota-Type Villemot and Pfeifer (2017) Prior		
Parameter	***Description***	***Chosen Value***
τ	Overall tightness	1
d	Speed at which lags greater than 1 decay	Not applicable since no lags
ω	Covariance dummies	1
λ	Co-persistence	No sum-of-coefficients
μ	Own-persistence	No dummy initial observations

Transition Probabilities: Dirichlet Prior		
Parameter	***Prior Mean***	***Prior Std. Dev.***
p_{ij}, $\forall i,j$ & $i \neq j$	0.2	0.2

Parameters in Endogenous Transition Probabilities				
Parameter	***Initial Guess***	***Prior Distribution***	***Prior Mean***	***Prior Std. Dev.***
b_{ij}, $\forall i,j$ & $i \neq j$	0	Normal	0	2
c_{ij}, $\forall i,j$ & $i \neq j$	0	Normal	0	2

[12]The toolbox was developed by Junior Maih and is available at https://github.com/jmaih/RISE_toolbox.

B.2 Two Regimes with Agnostic Transition Probabilities

Table B-2: Estimated Transition Probabilities $Pr(s_{t+1} = j | s_t = i)$

	s_{t+1} = Bad Regime		s_{t+1} = Normal Regime	
s_t = Bad Regime	0.95		0.05	[0.03,0.08]
s_t = Normal Regime	0.03	[0.02,0.04]	0.97	

Note: Values in brackets indicate lower (5%) and upper (95%) bound of coefficients derived from their posterior distribution calculated via MCMC using 20,000 replications, a burn-in of 1,000 replications (thus for a total of 210,000 effective replications). Regime-switching regression with two regimes and one Markov chain for both coefficients and volatility. Notice that the diagonal elements are not estimated since the state transition probabilities add up to one for a given regime.

Table B-3: Estimated Regime-Switching Coefficients.

	Average GDP Growth Over Next Year			
	Bad Regime		**Normal Regime**	
$\alpha_y(s_t)$	-0.97	[-1.20,-0.80]	0.57	[0.47, 0.66]
$\beta_y(s_t)$	-0.29	[-0.54,-0.13]	-0.02	[-0.07, 0.03]
$\gamma_y(s_t)$	0.51	[0.36, 0.71]	0.27	[0.19, 0.35]
$\sigma_y(s_t)$	2.62	[2.41, 2.84]	0.65	[0.59, 0.70]
	Financial Factor		**Macroeconomic Factor**	
$\alpha_{f/m}$	0.00	[0.00, 0.00]	0.00	[0.00, 0.00]
$\beta_{f/m}$	0.88	[0.85, 0.92]	-0.07	[-0.08,-0.05]
$\eta_{f/m}$	0.05	[-0.13, 0.20]	0.82	[0.79, 0.85]
γ_f	-0.10	[-0.26, 0.06]		
$\sigma_{f/m}$	0.63	[0.60, 0.67]	0.34	[0.32, 0.35]

Note: Values in brackets indicate lower (5%) and upper (95%) bound of coefficients derived from their posterior distribution calculated via MCMC using 20,000 replications, a burn-in of 1,000 replications (thus for a total of 210,000 effective replications). Regime-switching regression with two regimes and one Markov chain for both coefficients and volatility.

B.3 Two Regimes and f_t and m_t Exogenous

We consider the following regime-switching model

$$\bar{\Delta} y_{t+1,t+12} = \alpha_y(s_t) + \beta_y(s_t) f_t + \gamma_y(s_t) m_t + \sigma_y(s_t)\varepsilon_t^y, \quad \text{(B-1)}$$

where both the coefficients $\theta(s_t) \equiv \{\alpha(s_t), \beta(s_t), \gamma(s_t)\}$ and the standard deviation $\sigma(s_t)$ vary depending on an unobserved regime variable $s_t \in \{1,2\}$ which indicates the regime prevailing at time t. The latent variable s_t is governed by a discrete time, discrete state Markov stochastic process, defined by the transition probabilities:

$$p_{ij} \equiv Pr\left(s_{t+1} = j | s_t = i\right), \quad \sum_{j=1}^{2} p_{ij} = 1, \quad \forall i, j \in \{1,2\} \tag{B-2}$$

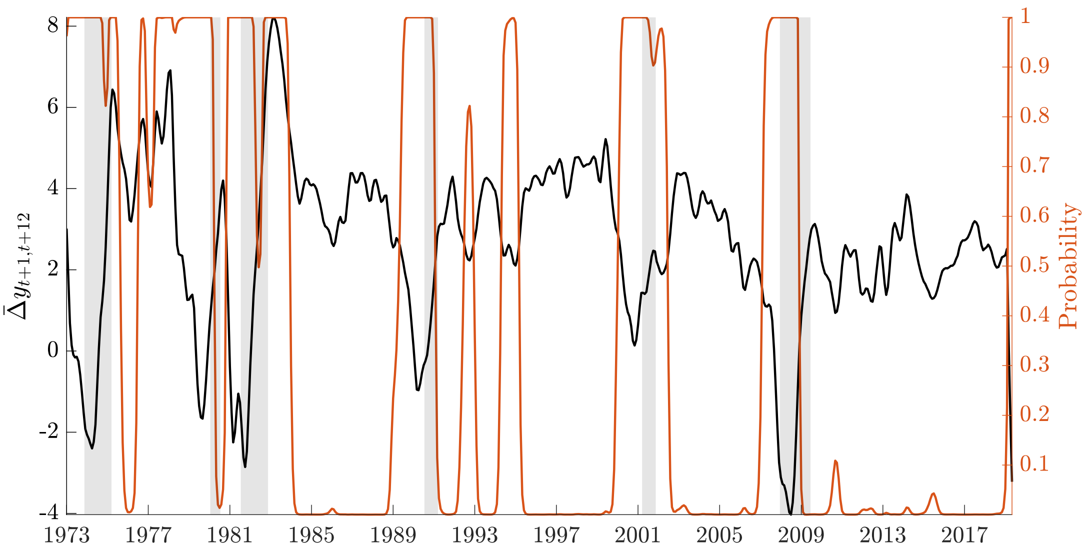
We estimate this regression using the same sample and methods as in the baseline model.

Table B-4: Estimated Regime-Switching Coefficients. Financial and Macroeconomic Factor are Exogenous.

	Average GDP Growth Over Next Year			
	Bad Regime		Normal Regime	
$\alpha_y(s_t)$	-0.97	[-1.32,-0.57]	0.57	[0.46, 0.67]
$\beta_y(s_t)$	-0.29	[-0.55,-0.04]	-0.02	[-0.08, 0.04]
$\gamma_y(s_t)$	0.51	[0.06, 1.02]	0.27	[0.13, 0.40]
$\sigma_y(s_t)$	2.62	[2.38, 2.75]	0.64	[0.58, 0.71]

NOTE: Values in brackets indicate lower (5%) and upper (95%) bound of coefficients derived from their posterior distribution calculated via MCMC using 20,000 replications, a burn-in of 1,000 replications (thus for a total of 210,000 effective replications). Regime-switching regression with two regimes and one Markov chain for both coefficients and volatility. Financial and macroeconomic factor are exogenous.

Figure B-1: Estimated "Bad" Regime Probabilities. Financial and Macroeconomic Factor are Exogenous.



Note: Estimated "bad" regime probabilities from Markov-switching regression (red) against average GDP growth over next year (black). Regime-switching regression with two regimes and one Markov chain for both coefficients and volatility. Financial and macroeconomic factor are exogenous.

Table B-5: Estimated Transition Probabilities $Pr\left(s_{t+1}=j|s_t=i\right)$. Financial and Macroeconomic Factor are Exogenous.

	s_{t+1} = Bad Regime		s_{t+1} = Normal Regime	
s_t = Bad Regime	0.95		0.05	[0.02,0.09]
s_t = Normal Regime	0.03	[0.01,0.05]	0.97	

Note: Values in brackets indicate lower (5%) and upper (95%) bound of coefficients derived from their posterior distribution calculated via MCMC using 20,000 replications, a burn-in of 1,000 replications (thus for a total of 210,000 effective replications). Regime-switching regression with two regimes and one Markov chain for both coefficients and volatility. Financial and macroeconomic factor are exogenous. Notice that the diagonal elements are not estimated since the state transition probabilities add up to one for a given regime.

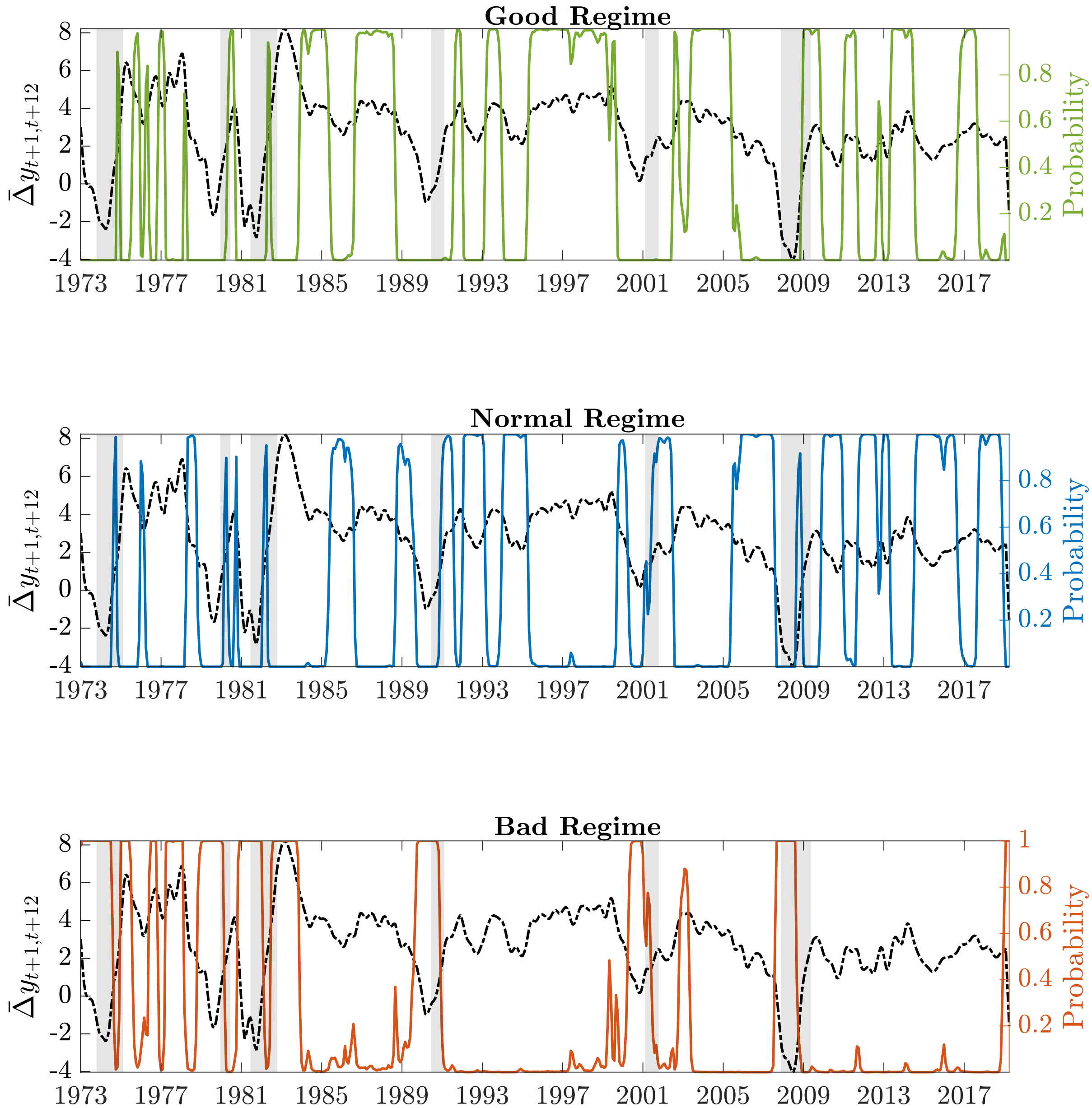
B.4 Three Regimes and One Endogenous Markov Chain for Coefficients and Volatility.

Table B-6: Estimated Endogenous Transition Probabilities.

p_{12} from Good to Normal Regime			p_{13} from Good to Bad Regime		
b_{12}	-0.35	[-0.53,-0.14]	b_{13}	2.69	[2.12, 3.02]
c_{12}	-0.29	[-0.43,-0.18]	c_{13}	0.37	[0.27, 0.61]
p_{21} from Normal to Good Regime			p_{23} from Normal to Bad Regime		
b_{21}	0.45	[0.27, 0.57]	b_{23}	3.27	[2.96, 3.56]
c_{21}	-2.71	[-4.07,-2.19]	c_{23}	0.90	[0.76, 1.06]
p_{31} from Bad to Good Regime			p_{32} from Bad to Normal Regime		
b_{31}	-0.89	[-0.97,-0.72]	b_{32}	2.52	[2.26, 2.66]
c_{31}	2.71	[2.50, 2.99]	c_{32}	-0.95	[-1.09,-0.82]

NOTE: Values in brackets indicate lower (5%) and upper (95%) bound of coefficients derived from their posterior distribution calculated via MCMC using 20,000 replications, a burn-in of 1,000 replications (thus for a total of 210,000 effective replications). Regime-switching regression with three regimes and one Markov chain for both coefficients and volatility. Transition probabilities are endogenous and depend both on the financial and the macroeconomic factor. Notice that the diagonal elements are not estimated since the state transition probabilities add up to one for a given regime.

Figure B-2: Regime Probabilities for Markov-Switching Regression with Three Regimes and One Markov Chain for Both Coefficients and Volatility.



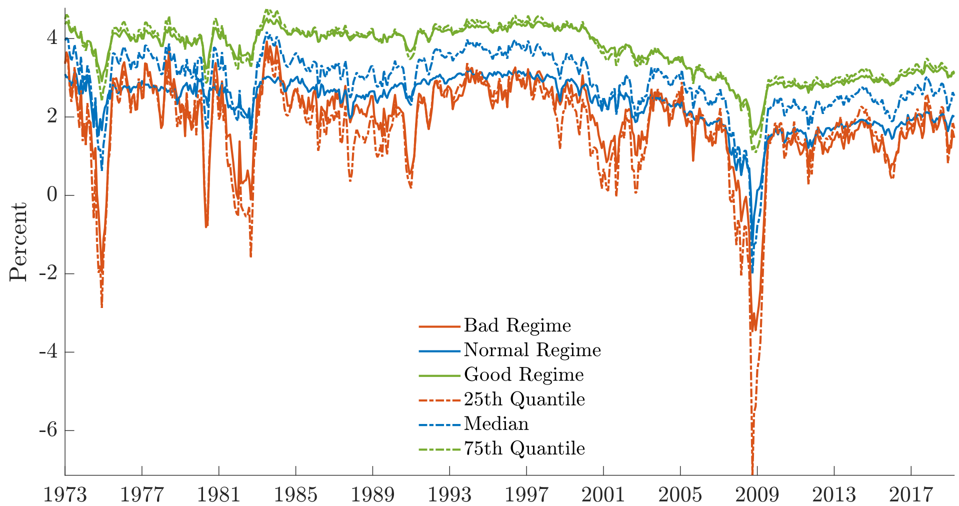
NOTE: Estimated regime probabilities from Markov-switching regression against average GDP growth over next year (black). Markov-switching regression with three regimes and one Markov chain for both coefficients and volatility. Transition probabilities are endogenous and depend both on the financial and the macroeconomic factor.

Table B-7: Estimated Coefficients of Alternative Regime Switching and Quantile Regression Models

	Regime-Switching					
	Three Regimes and One Markov Chain for Both Coefficients and Volatility					
	Bad Regime		***Normal Regime***		***Good Regime***	
$\alpha_y(s_t)$	-0.79	[-0.99,-0.60]	-0.33	[-0.38,-0.29]	0.98	[0.93, 1.02]
$\beta_y(s_t)$	-0.24	[-0.32,-0.12]	-0.25	[-0.28,-0.21]	-0.07	[-0.09,-0.04]
$\gamma_y(s_t)$	0.97	[0.88, 1.09]	0.03	[-0.05, 0.11]	0.24	[0.19, 0.28]
$\sigma_y(s_t)$	3.38	[3.27, 3.49]	0.47	[0.43, 0.50]	0.33	[0.31, 0.35]
	Quantile Regression					
	25th Quantile		***Median***		***75th Quantile***	
α_τ	-0.99	[-1.11,-0.86]	0.20	[0.11, 0.29]	1.02	[0.96, 1.08]
β_τ	-0.60	[-0.71,-0.48]	-0.29	[-0.37,-0.21]	-0.09	[-0.14,-0.04]
γ_τ	0.68	[0.42, 0.94]	0.38	[0.21, 0.55]	0.33	[0.20, 0.46]

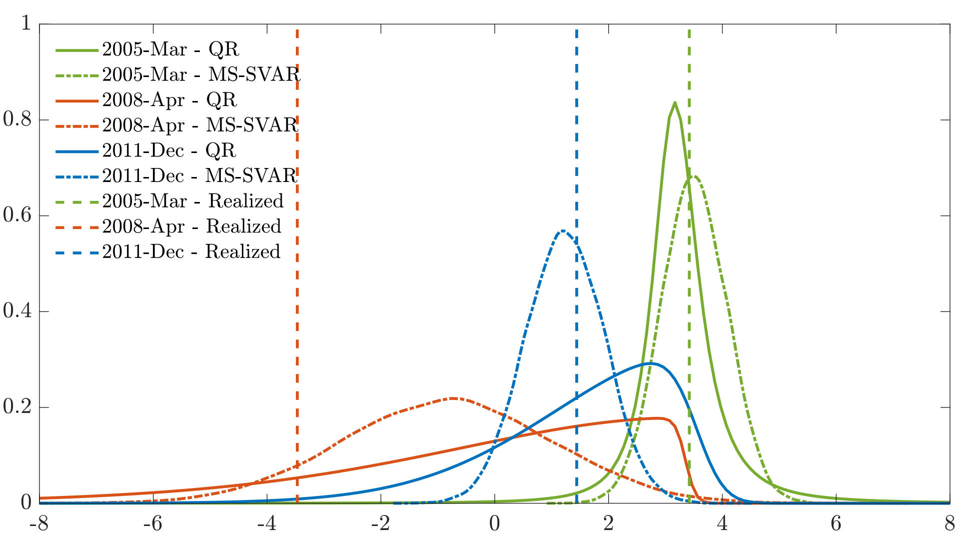
NOTE: Values in brackets for regime-switching models indicate lower (5%) and upper (95%) bound of coefficients derived from their posterior distribution calculated via MCMC using 20,000 replications, a burn-in of 1,000 replications (thus for a total of 210,000 effective replications). Values in brackets for regime-switching models indicate lower (16%) and upper (84%) bound of coefficients computed via "blocks-of-blocks" bootstrap (see Appendix C) using 500 replications.

Figure B-3: Fitted Values: Alternative Regime-Switching vs. Quantile Regression.



Note: The figure reports regime-specific fitted values of average GDP growth over the next year along with the conditional quantiles of that same variable estimated from the quantile regression. The model features one Markov chain governing both coefficients and volatility. Transition probabilities are endogenous and depend both on the financial and the macroeconomic factor.

Figure B-4: Densities: Alternative Regime-Switching vs. Quantile Regression.



Note: Densities of average GDP growth over the next year from Markov-switching regression and from skewed-t approximation of estimated conditional quantiles from quantile regression. The model features one Markov chain governing both coefficients and volatility. Transition probabilities are endogenous and depend both on the financial and the macroeconomic factor.

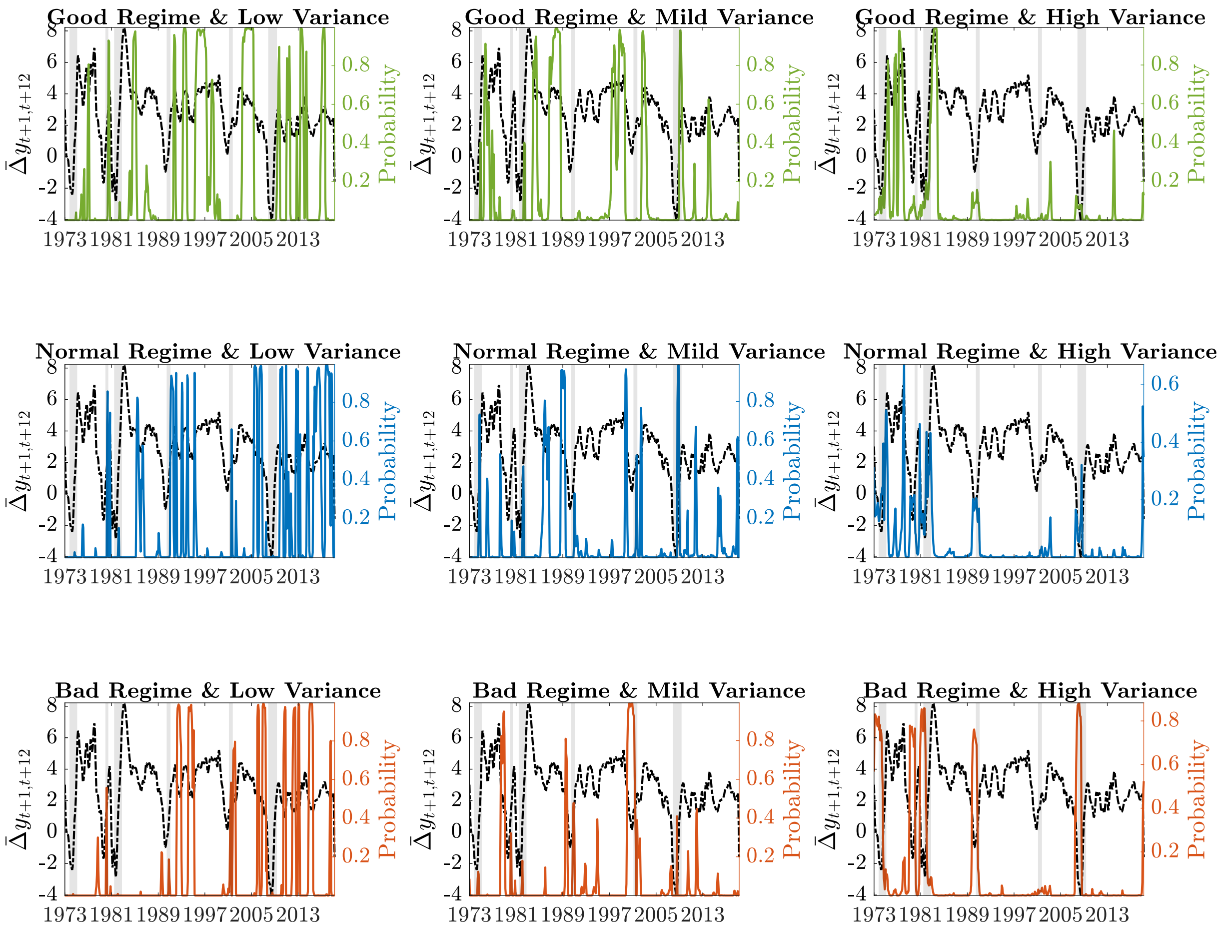
B.5 Three Regimes and Separate Endogenous Markov Chains for Coefficients and Volatility

Table B-8: Estimated Endogenous Transition Probabilities.

	p_{12} from Good to Normal		q_{12} from Low to Mild Volatility	
b_{12}	0.03	[-0.72, 0.69]	2.66	[1.40, 4.45]
c_{12}	-1.98	[-4.07, 0.20]	1.07	[-2.13, 4.01]
	p_{13} from Good to Bad		q_{13} from Low to High Volatility	
b_{13}	1.06	[-0.58, 3.51]	4.00	[2.35, 6.24]
c_{13}	-4.67	[-7.45,-1.71]	1.72	[-1.24, 3.86]
	p_{21} from Normal to Good		q_{21} from Mild to Low Volatility	
b_{21}	-0.36	[-1.28, 0.53]	-2.05	[-4.27,-0.28]
c_{21}	-1.40	[-3.63, 0.41]	-1.61	[-3.76, 0.88]
	p_{23} from Normal to Bad		q_{23} from Mild to High Volatility	
b_{23}	0.04	[-1.09, 1.05]	-1.92	[-3.79,-0.56]
c_{23}	1.01	[-0.38, 2.70]	-2.06	[-4.31, 0.47]
	p_{31} from Bad to Good		q_{31} from High to Low Volatility	
b_{31}	2.72	[-0.14, 5.01]	-0.65	[-2.29, 1.35]
c_{31}	-0.40	[-4.25, 2.62]	0.00	[-2.64, 4.67]
	p_{32} from Bad to Normal		q_{32} from High to Mild Volatility	
b_{32}	-0.73	[-2.41, 0.37]	0.16	[-1.18, 2.61]
c_{32}	-1.36	[-4.07, 0.89]	0.52	[-1.80, 4.10]

NOTE: Values in brackets indicate lower (5%) and upper (95%) bound of coefficients derived from their posterior distribution calculated via MCMC using 20,000 replications, a burn-in of 1,000 replications (thus for a total of 210,000 effective replications). Regime-switching regression with three regimes and independent Markov chains for coefficients and volatility. Transition probabilities are endogenous and depend both on the financial and the macroeconomic factor. Notice that the diagonal elements are not estimated since the state transition probabilities add up to one for a given regime.

Figure B-5: Regime Probabilities for Markov-Switching Regression with Three Regimes and Separate Markov Chains for Coefficients and Volatility.



NOTE: Estimated regime probabilities from Markov-switching regression against average GDP growth over next year (black). Markov-switching regression with three regimes and independent Markov chains for coefficients and volatility. Transition probabilities are endogenous and depend both on the financial and the macroeconomic factor.

C Details on Quantile Regression

C.1 Azzalini and Capitanio (2003) Procedure

We follow the approacho proposed by Azzalini and Capitanio (2003) of smoothing the quantile function using the skewed $t-$distribution. This flexible distribution is characterized by four parameters:

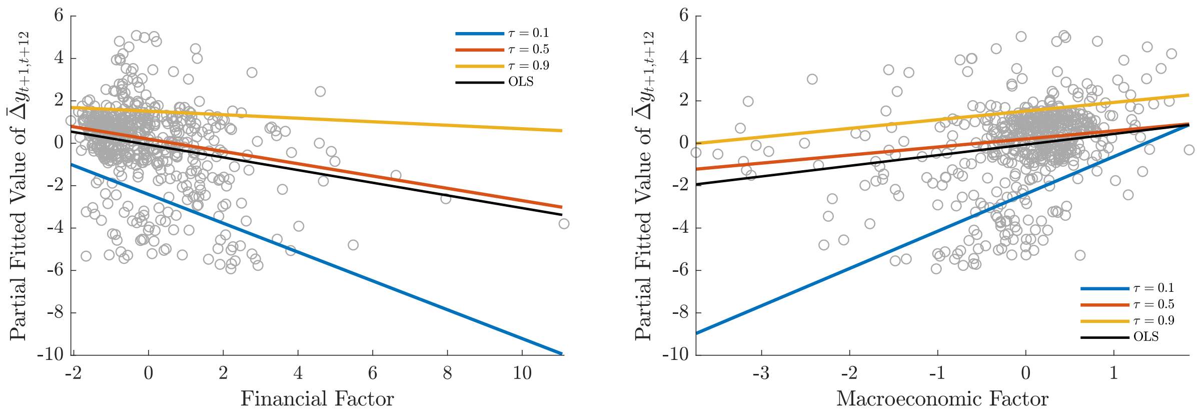
$$f(\bar{\Delta}y_{t+1,t+12}|x_t,\mu_t,\sigma_t,\eta_t,\kappa_t) = \frac{2}{\sigma_t} \times t\,(z_{t,t+4};\kappa_t) \times T\left(\eta_t\, z_{t,t+4}\sqrt{\frac{\kappa_t+1}{\kappa_t+z_{t,t+4}^2}};\kappa_t+1\right), \quad \text{(C-1)}$$

where $z_{t,t+4} = \frac{y_{t+1,t+12}(x_t)-\mu_t}{\sigma_t}$ and t and T respectively represent the density and cumulative distribution function of the student t-distribution. The constants $\mu_t \in \mathbb{R}$ and $\sigma_t \in \mathbb{R}^+$ are location and scale parameters, whereas the constants $\eta_t \in \mathbb{R}$ and $\kappa_t \in \mathbb{Z}^+$ control the skewness and the kurtosis of the distribution, respectively. As in Adrian *et al.* (2019), we compute these parameters at each point in time t to minimize the squared distance between our estimated quantile function $\widehat{\mathcal{Q}}_\tau(\bar{\Delta}y_{t+1,t+12}|x_t)$, obtained from the quantile Phillips-curve model (16), and the quantile function of the skewed $t-$distribution $F^{-1}(\bar{\Delta}y_{t+1,t+12}|x_t,\mu_t,\sigma_t,\eta_t,\kappa_t)$ to match the 10$^{\text{th}}$, 25$^{\text{th}}$, 75$^{\text{th}}$ and 90$^{\text{th}}$ quantiles.

C.2 Slopes and Conditional Distributions

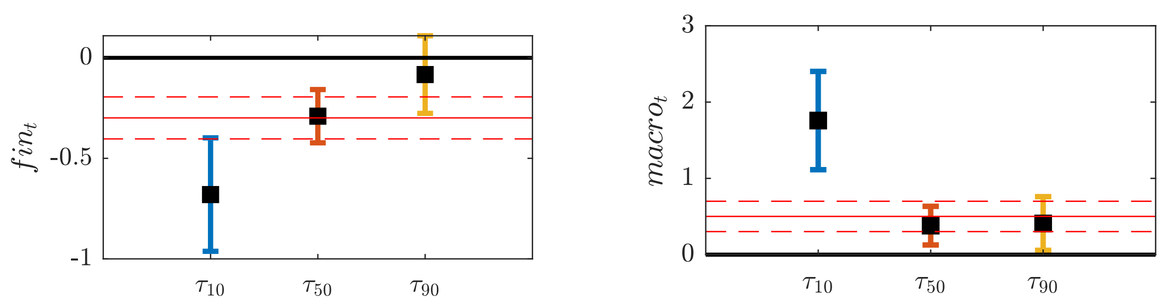
Figure C-1: Quantile Regression Slopes.

Point Estimates



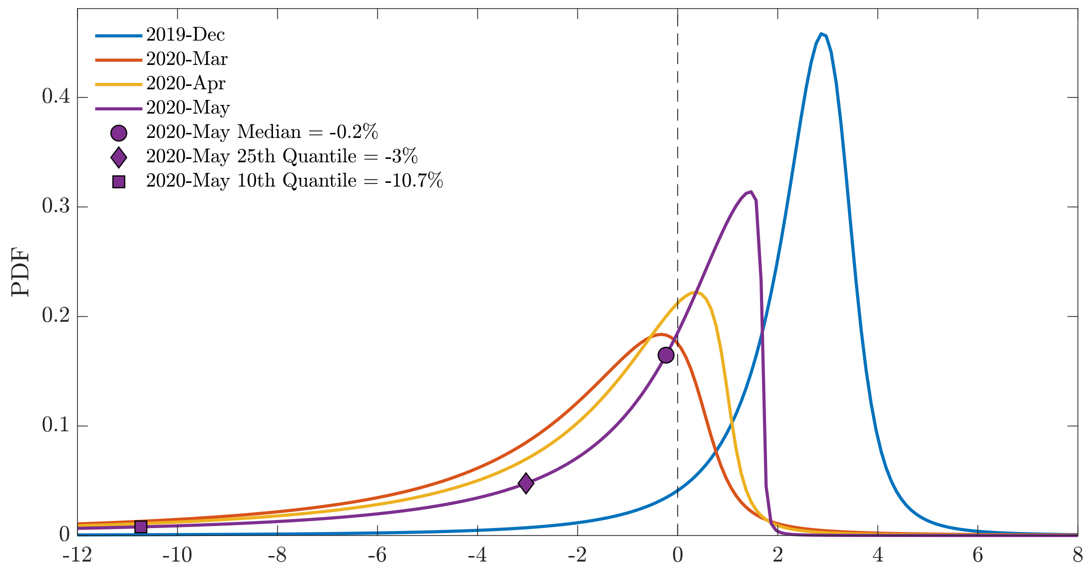
NOTE: Slope coefficients of the quantile regression of average GDP growth over the next year. The lines illustrate the slopes associated with the median (red), the 10$^{\text{th}}$ (blue) and the 90$^{\text{th}}$ (yellow) quantile. The black lines are the OLS estimates. Circles indicate scatterplots of average future GDP growth against the conditioning variable.

Confidence Intervals



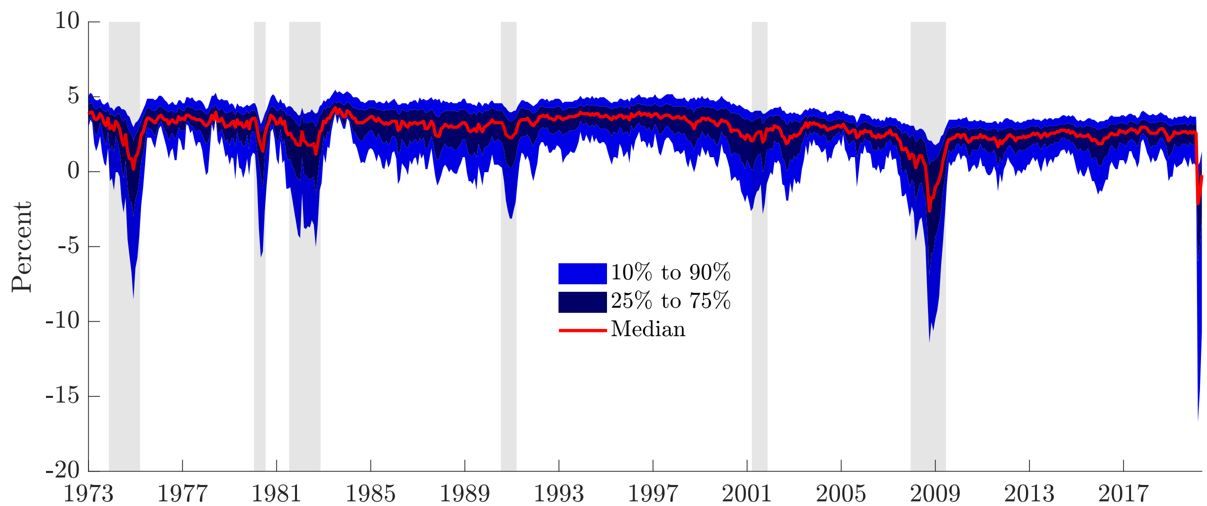
NOTE: The black squares correspond to the point estimates whereas the vertical lines to the 68% confidence intervals computed via "blocks-of-blocks" bootstrap using 500 replications for the 10$^{\text{th}}$ quantile (blue), median (red) and 90$^{\text{th}}$ quantile (yellow). The OLS estimates and their 90% confidence intervals are represented by the red lines.

Figure C-2: Conditional Distribution of Average GDP Growth over the Next Year.



NOTE: Selected conditional distributions of average GDP growth over the next year, obtained by fitting skewed-t distribution on conditional quantiles obtained from quantile regression.

Figure C-3: Conditional Quantiles of Average GDP Growth over the Next Year.



NOTE: Time evolution of conditional quantiles associated with fitted skewed-t distribution of average GDP growth over the next year.

D Robustness to Other Conditioning Variables

We explore the uncertainty around our quantile coefficient estimates as well as the robustness of our analysis to alternative choices of conditioning variables in the quantile regression. In particular, Figure D-1 explores different measures of the financial factor (ECB and Kansas City FRB financial factors) and of the macroeconomic factor (Phildalphia FED ADS Index. Overall, all specifications point to the same qualitative patterns picked up by our baseline specification. Figure D-2 explores the sensitivity of our results to the use of the Chicago FED National Financial Conditions Index and to its subcomponents (leverage, risk and credit) instead of the financial factor estimated from our DFM model. Finally, Figure D-3 explores how our results change if we (i) use GDP growth instead of the macroeconomic factor - in this case the quantile regression reveals that there is no nonlinear relationship between current and future economic conditions as the estimated regression coefficients are the same across quantiles, (ii) use the (monthly) version of our model that follows Adrian *et al.* (2019) by having the NFCI and GDP growth as conditioning variables - this case is similar to (i), (iii) do not detrend average GDP growth over the next year (our dependent variable) - results are similar to the baseline, and (iv) if we include in the regression a second financial factor obtained from the monthly DFM model.

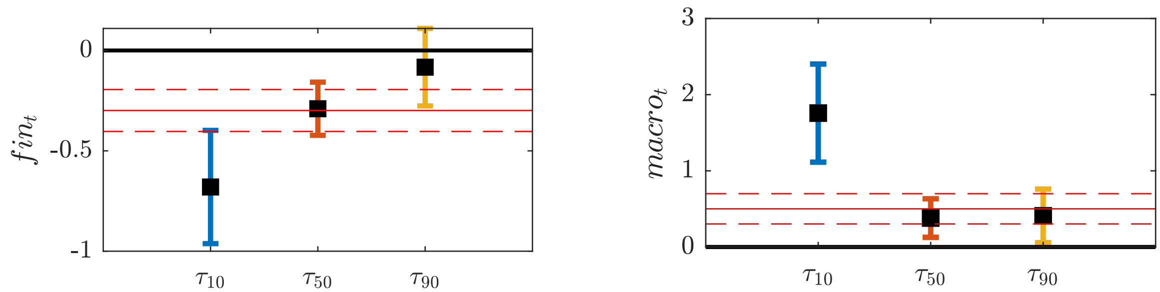
Bootstrap Procedure To compute confidence bands for the quantile regression model we revert to "blocks-of-blocks" bootstrap. While more details on this methodology can be found in Kilian and Lütkepohl (2018) (see Chapter 12 therein), we here provide a brief summary of the bootstrap procedure.

"Blocks-of-blocks" bootstrap is used in cases where a researcher is interested in computing confidence intervals around nonsymmetric statistics of the underlying data (e.g., autocorrelations or estimators of autoregressive slope coefficients in a time-series context). This is relevant in our case since not only the quantile regression slopes are non-linear functions of the data but also, we are de facto running a h-step predictive regression of inflation on its (past) determinants. The "blocks-of-blocks" bootstrap procedure allows to preserve the (time-series) dependency in the data, which would in most cases be destroyed by a naive bootstrap.

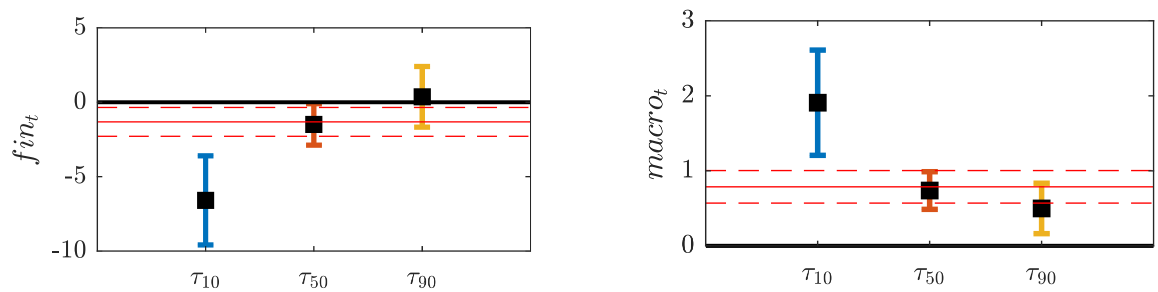
More specifically, the "blocks-of-blocks" bootstrap procedure relies on first dividing the dependent variable y and the regressors X into consecutive blocks of all possible m-tuples. At each bootstrap replication, blocks of data are randomly drawn to form a new sample of the same size as the original data. Importantly, the blocks are resampled in the same order for both the dependent variable y and the regressors X, a key step which preserves the time-dependency in the data. In our particular application, we run the quantile regression and store the estimates corresponding to each bootstrap replication. From the distribution of these estimates, 68 percent confidence intervals are constructed and centered around the point estimate obtained with the original sample. The procedure is asymptotically valid for stationary processes if the block size l increases at a suitable rate as $T \to \infty$. Following Berkowitz *et al.* (1999) we set $m = \sqrt[3]{T}$, where T is the sample size. Finally, this bootstrap procedure preserves the quantile regression feature of being agnostic about the underlying distribution of the error terms, as this is not a residual-based procedure.

Figure D-1: Quantile Regression Slopes Across Specifications (I).

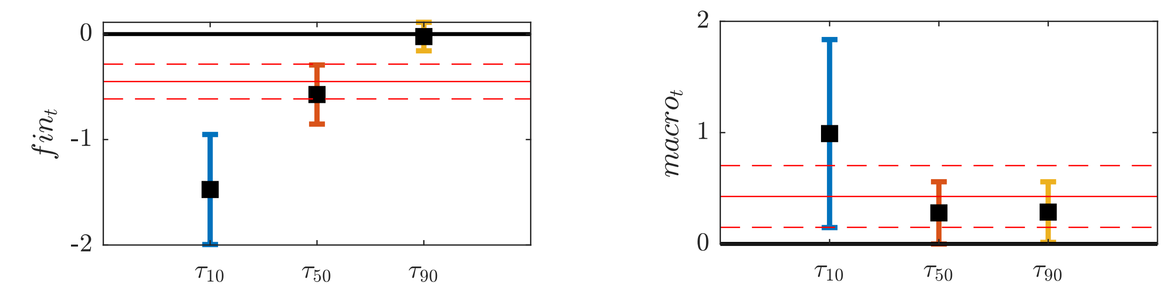
Baseline



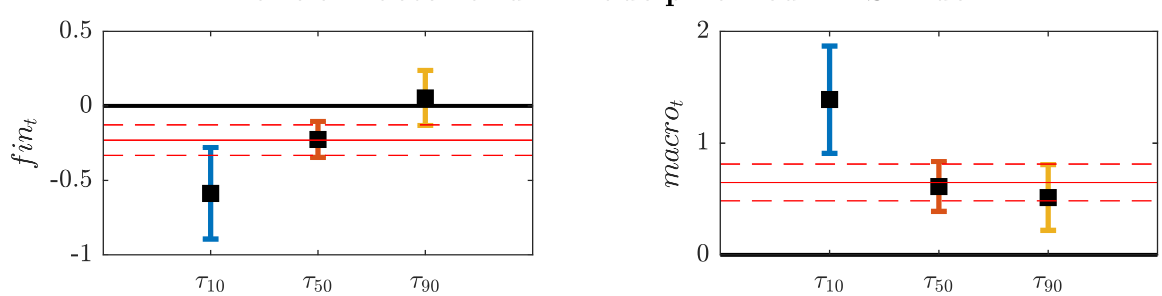
ECB Financial Factor (starts in January 1980)



Kansas City Fed Financial Factor (starts in February 1990)



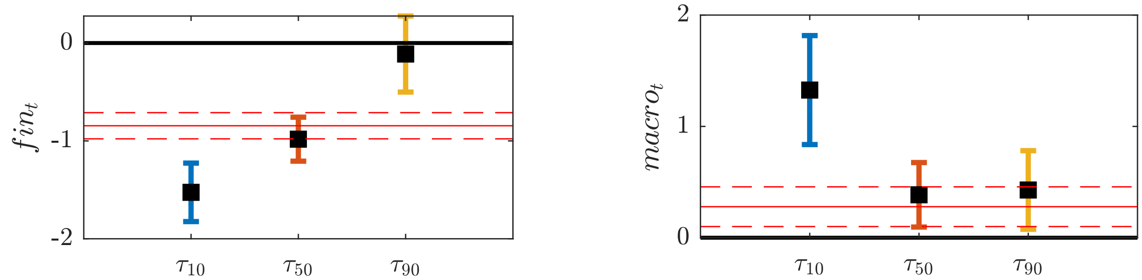
Financial Factor and Philadelphia Fed ADS Index



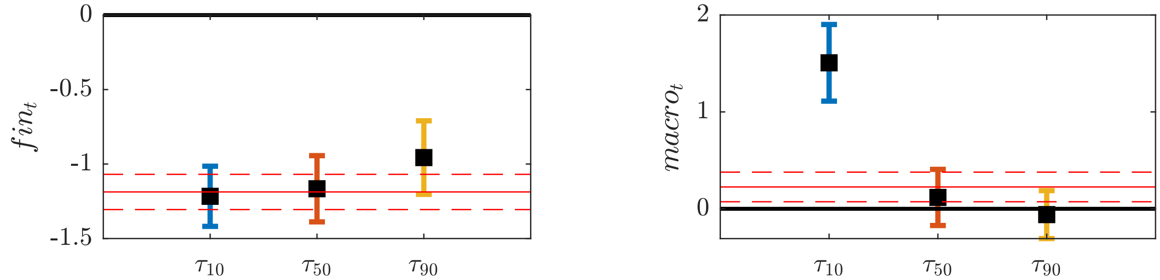
NOTE: The black squares correspond to the point estimates whereas the vertical lines to the 68% confidence intervals computed via "blocks-of-blocks" bootstrap using 500 replications for the 10^{th} quantile (blue), median (red) and 90^{th} quantile (yellow). The OLS estimates and their 90% confidence intervals are represented by the red lines.

Figure D-2: Quantile Regression Slopes Across Specifications (II).

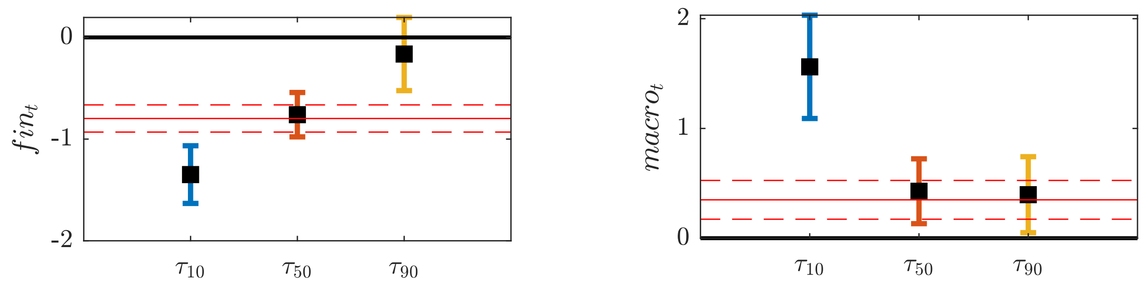
Chicago FED National Financial Conditions Index (NFCI)



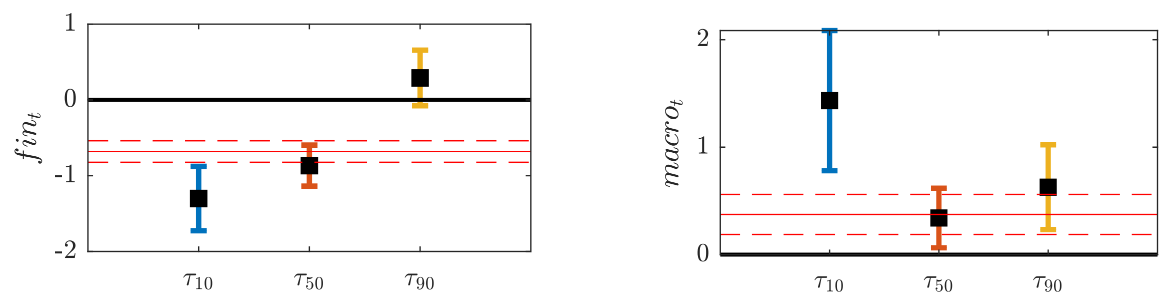
NFCI Leverage Component



NFCI Risk Component



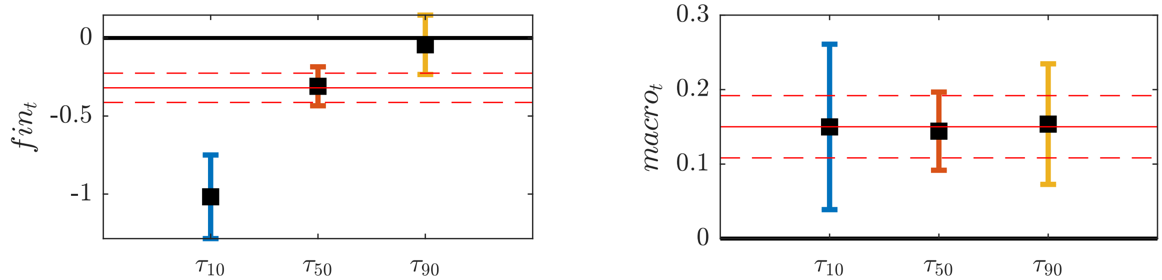
NFCI Credit Component



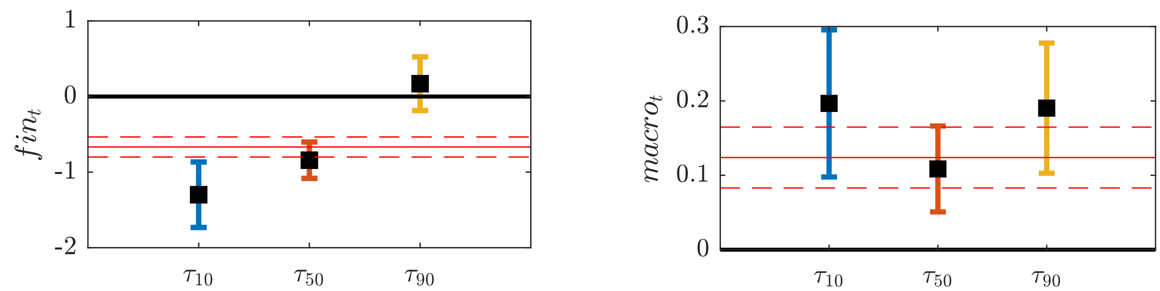
NOTE: The black squares correspond to the point estimates whereas the vertical lines to the 68% confidence intervals computed via "blocks-of-blocks" bootstrap using 500 replications for the 10^{th} quantile (blue), median (red) and 90^{th} quantile (yellow). The OLS estimates and their 90% confidence intervals are represented by the red lines.

Figure D-3: Quantile Regression Slopes Across Specifications (III).

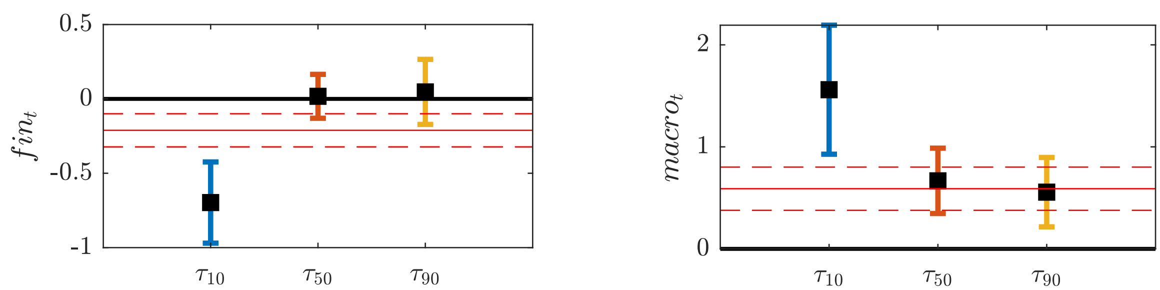
Financial Factor and GDP Growth



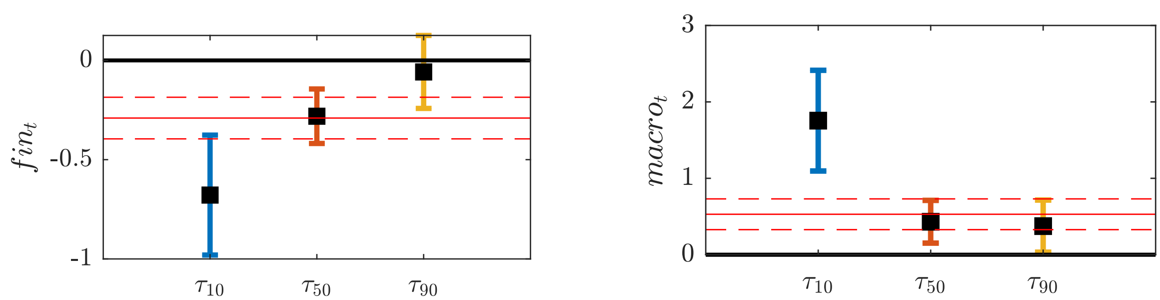
NFCI and GDP Growth



Baseline without Detrending GDP Growth



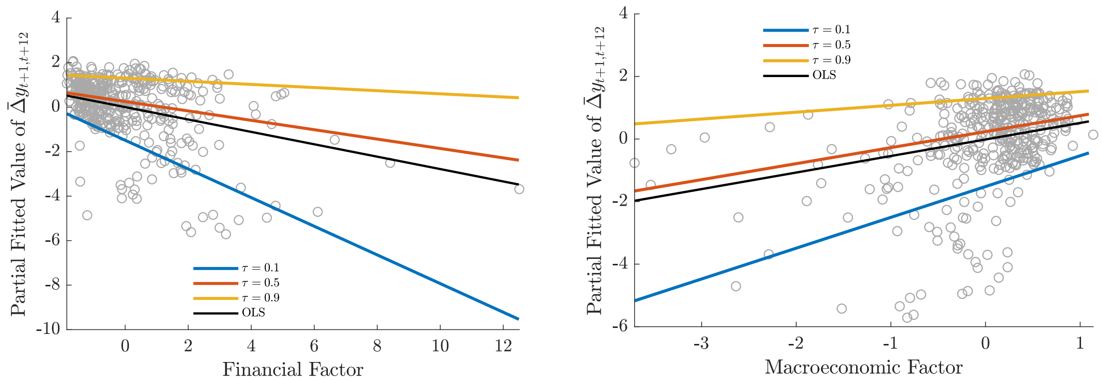
Financial Factor I and Financial Factor 2



NOTE: The black squares correspond to the point estimates whereas the vertical lines to the 68% confidence intervals computed via "blocks-of-blocks" bootstrap using 500 replications for the 10^{th} quantile (blue), median (red) and 90^{th} quantile (yellow). The OLS estimates and their 90% confidence intervals are represented by the red lines.

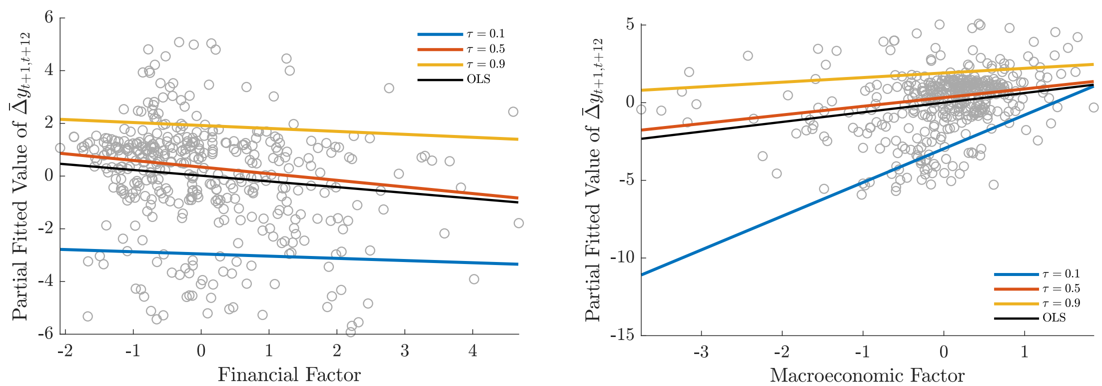
D.1 Subsamples

Figure D-4: Starting in 1986.



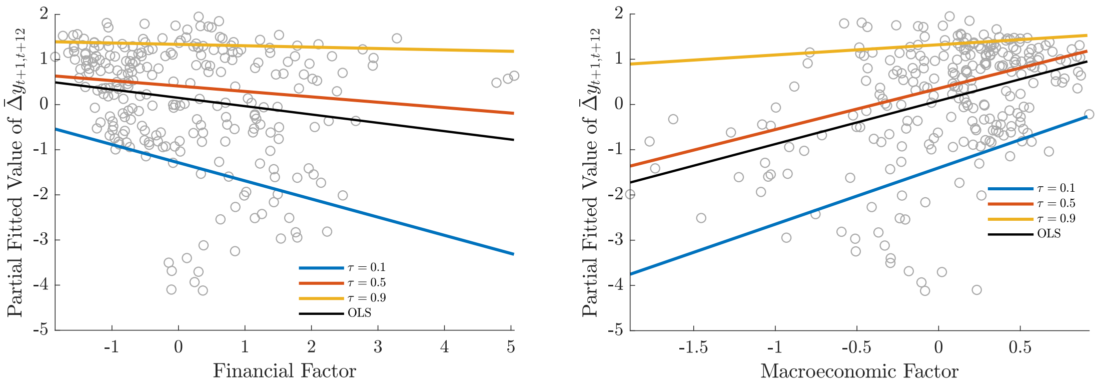
NOTE: The figure displays the slope coefficients of the quantile regression of average GDP growth over the next year starting in January 1986. The lines illustrate the slopes associated with the median (red), the 10$^{\text{th}}$ (blue) and the 90$^{\text{th}}$ (yellow) quantile. The black lines are the OLS estimates. Circles indicate scatterplots of average future GDP growth against the conditioning variable.

Figure D-5: Stopping Before the Great Recession.



NOTE: The figure displays the slope coefficients of the quantile regression of average GDP growth over the next year starting in January 1973 and stopping in December 2007. The lines illustrate the slopes associated with the median (red), the 10$^{\text{th}}$ (blue) and the 90$^{\text{th}}$ (yellow) quantile. The black lines are the OLS estimates. Circles indicate scatterplots of average future GDP growth against the conditioning variable.

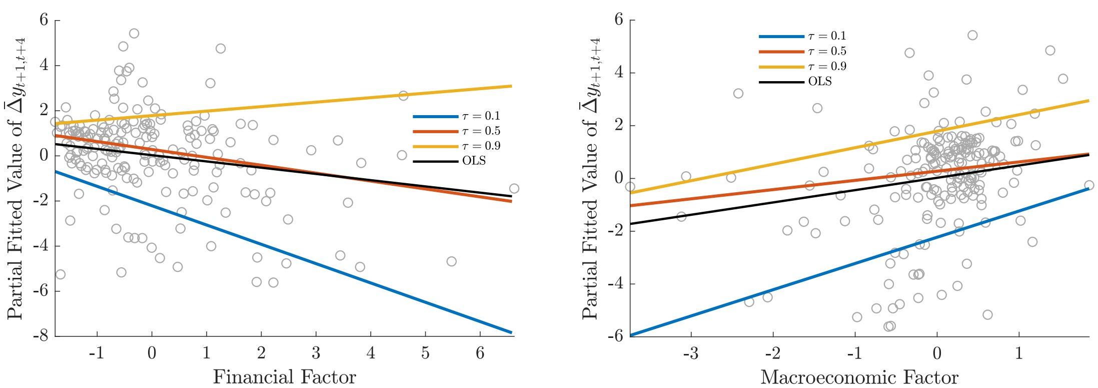
Figure D-6: Starting in 1986 and stopping Before the Great Recession.



NOTE: The figure displays the slope coefficients of the quantile regression of average GDP growth over the next year starting in January 1986 and stopping in December 2007. The lines illustrate the slopes associated with the median (red), the 10th (blue) and the 90th (yellow) quantile. The black lines are the OLS estimates. Circles indicate scatterplots of average future GDP growth against the conditioning variable.

D.2 Quarterly Model with BEA GDP Growth

Figure D-7: Quantile Regression Slopes.

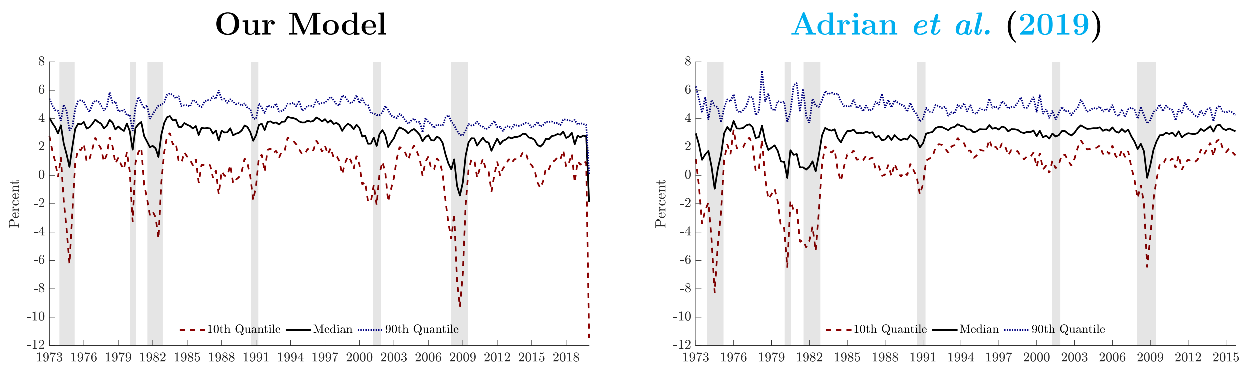


NOTE: The figure displays the slope coefficients of the quantile regression of average BEA GDP growth over the next year in the quarterly specification of the model. The lines illustrate the slopes associated with the median (red), the 10th (blue) and the 90th (yellow) quantile. The black lines are the OLS estimates. Circles indicate scatterplots of average future GDP growth against the conditioning variable.

D.3 Comparison with Adrian *et al.* (2019)

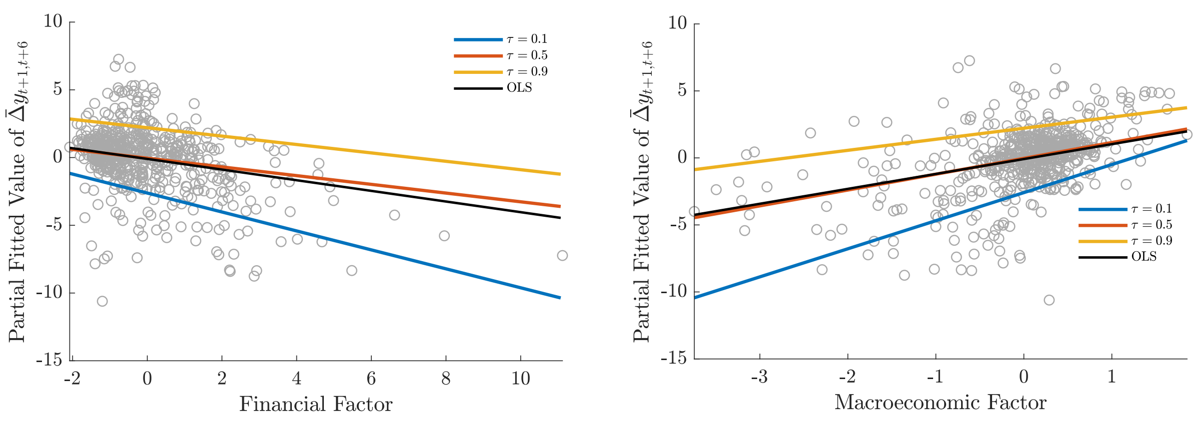
In Figure D-8 (left panel) we show the time evolution of the future GDP growth distribution by reporting the fitted values for the 10th quantile (left tail), the median and the 90th quantile (right tail) from our model estimated at quarterly frequency from 1973:Q1 to 2019:Q, which are similar to those obtained by Adrian *et al.* (2019) from 1973:Q1 to 2015:Q4 by conditioning on the National Financial Conditions Index (NFCI) and GDP growth (right panel).

Figure D-8: Quantiles of Average GDP Growth over the Next Year.



D.4 Average GDP Growth over the Next Six Months

Figure D-9: Quantile Regression Slopes.



NOTE: The figure illustrates slope coefficients of the quantile regression of average GDP growth over the next half-year. The lines illustrate the slopes associated with the median (red), the 10^{th} (blue) and the 90^{th} (yellow) quantile. The black lines are the OLS estimates. Circles indicate scatterplots of average future GDP growth against the conditioning variable.

D.5 Out-of-Sample Analysis

Correct Calibration We now formally test for correct calibration of the conditional predictive distributions implied by our model. To do so we use the test of Rossi and Sekhposyan (2019), which evaluates the absolute predictive ability of a model at its estimated parameter values and, thus, in finite samples.[13] In this sense, both the parametric model and the estimation technique employed are being evaluated.

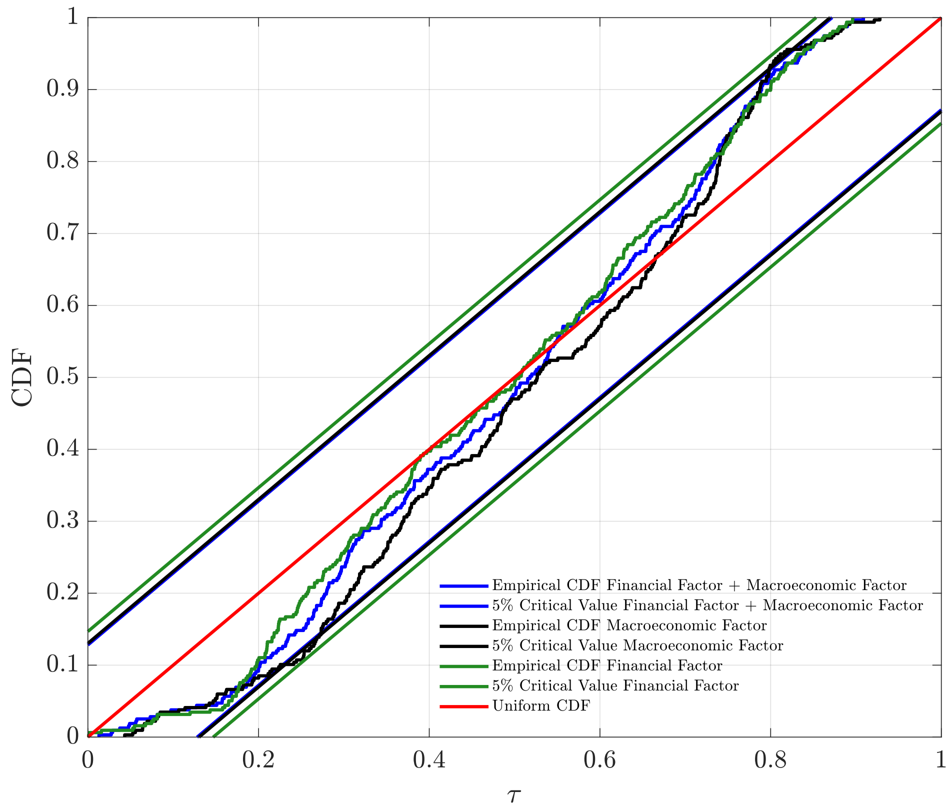
To run the test, we first define the probability integral transform (PIT), i.e., the conditional quantile z_t that corresponds to the realized observation $\bar{\Delta} y_{t+1,t+12}$:

$$z_t \equiv F^{-1}\left(\bar{\Delta} y^*_{t+1,t+12}|x_t\right) = Prob\left(\bar{\Delta} y_{t+1,t+12} < \bar{\Delta} y^*_{t+1,t+12}|x_t\right), \tag{D-1}$$

where $F^{-1}\left(\bar{\Delta} y^*_{t+1,t+12}|x_t\right)$ refers to the inverse of the conditional CDF or, equivalently, to the conditional quantile function evaluated at the realized value $\bar{\Delta} y_{t+1,t+12}$. In a perfectly calibrated model, the predictive density should feature a CDF which is uniform, i.e., equal to the 45° line. This property implies that the probability that the realized value is above or below the predicted value is the same (on average, across time) irrespectively of whether high or low realizations of the predicted variable are considered. Following this logic, if the empirical CDF of the PITs lies outside of the 5% critical values, then the Rossi and Sekhposyan (2019) rejects the null hypothesis of correct calibration.

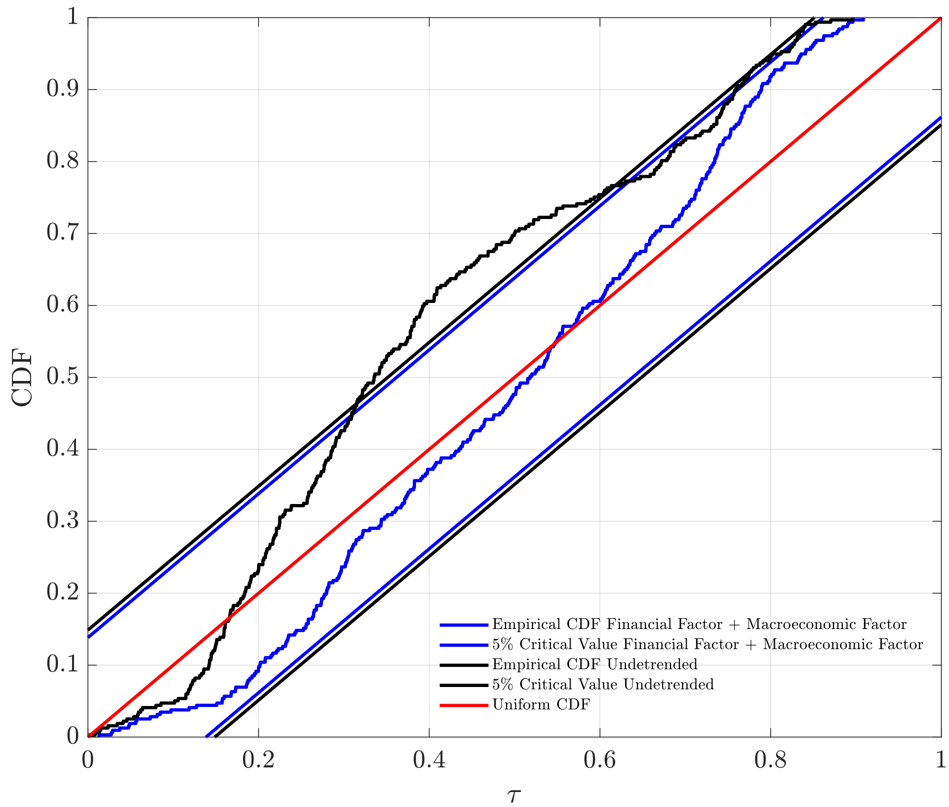
[13] See Rossi (2014) for an excellent summary of density forecast evaluations.

Figure D-10: Test for Correct Calibration of Predictive Density.



NOTE: The figure illustrates the CDF of a uniform distribution along with the empirical CDFs of out-of-sample PITs. The PITs are obtained from the density constructed by fitting a skewed-t distribution on the conditional quantiles from the quantile regression model (16). The 5% critical values for each model (dashed-dotted), are bootstrapped following the Rossi and Sekhposyan (2019) procedure for multi-step-ahead forecasts. As in Adrian *et al.* (2019), the PITs are constructed via an expanding rolling windows estimation of the quantile regression model (16) initially using 20 years of data. Confidence bands should thus be taken as general guidance since Rossi and Sekhposyan (2019) derive them for PITs computed using a fixed rolling window scheme.

Figure D-11: The Importance of Detrending for Correct Calibration.

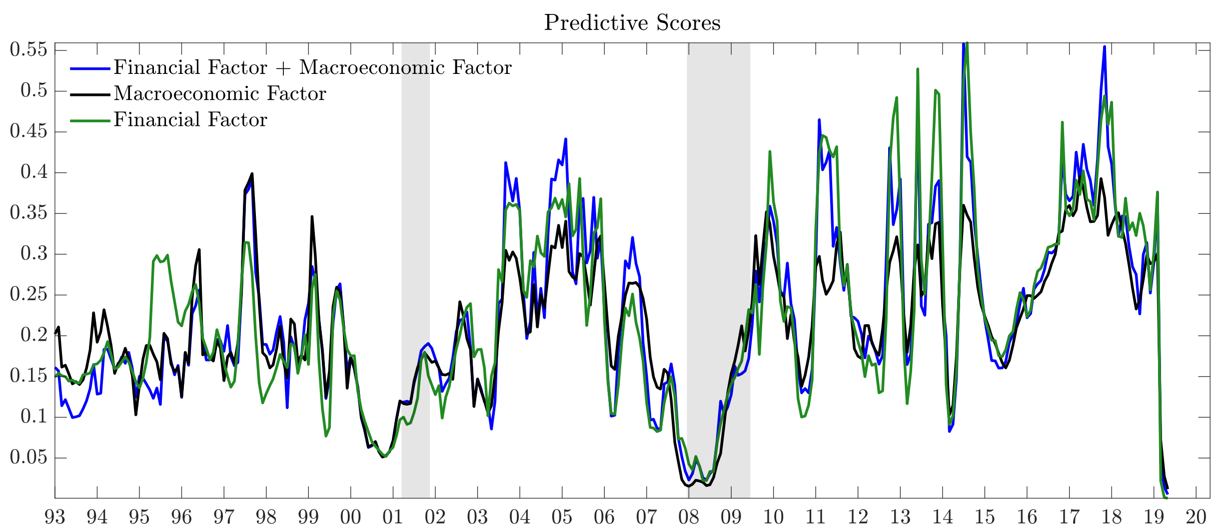


NOTE: The figure illustrates the CDF of a uniform distribution along with the empirical CDFs of out-of-sample PITs. The PITs are obtained from the density constructed by fitting a skewed-t distribution on the conditional quantiles from the quantile regression model (16). The 5% critical values for each model (dashed-dotted), are bootstrapped following the Rossi and Sekhposyan (2019) procedure for multi-step-ahead forecasts. As in Adrian *et al.* (2019), the PITs are constructed via an expanding rolling windows estimation of the quantile regression model (16) initially using 20 years of data. Confidence bands should thus be taken as general guidance since Rossi and Sekhposyan (2019) derive them for PITs computed using a fixed rolling window scheme.

Predictive Ability We further evaluate the reliability of the predictive distribution by measuring the accuracy of the model's density forecasts through its predictive scores. These are computed by evaluating the model's predictive distribution at the realized value of the time series. A higher the predictive score indicates more accurate predictions, as the model assigns a higher probability to outcomes that are closer to the realized value. We compute the predictive scores in an out-of-sample exercise as the previous one, where the predictive distributions are calculated using an expanding window with initial 20 years of data.

Figure D-12 plots the scores of the predictive distribution conditional on the financial and macroeconomic factor, our baseline model, together with the scores of the predictive distribution conditional on either financial or macroeconomic conditions alone.

Figure D-12: Predictive Scores.



NOTE: The figure illustrates the out-of-sample predictive scores obtained from the density constructed by fitting a skewed-t distribution on the conditional quantiles from the quantile regression model (16). In particular, the quantiles are constructed via an expanding rolling windows estimation initially using 20 years of data.

E FOMC Statements

- **Statement September 18, 2007**: *Today's action is intended to help forestall some of the adverse effects on the broader economy that might otherwise arise from the disruptions in financial markets and to promote moderate growth over time.*

- **Statement January 30, 2008**: *Financial markets remain under considerable stress, and credit has tightened further for some businesses and households. [...] Today's policy action, combined with those taken earlier, should help to promote moderate growth over time and to mitigate the risks to economic activity. However, downside risks to growth remain.*

- **Statement March 18, 2008**: *Financial markets remain under considerable stress [...] Today's policy action, combined with those taken earlier, including measures to foster market liquidity, should help to promote moderate growth over time and to mitigate the risks to economic activity. However, downside risks to growth remain.*