8th Forecasting Summer School
Forecasting High-Frequency Seasonal Time Series

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1 Course description

Seasonality is one of the most prominent features of time series observed at sub-annual frequencies. Advances in information technology and data collection have led to a surge in intra-daily, daily, and weekly time series, which require specialized modeling techniques.

Modelling and forecasting high-frequency seasonal time series poses several important challenges. First, the seasonal periods associated with annual and monthly cycles are often neither constant nor integer-valued. Secondly, the time series display complex seasonal patterns that result from the combination of multiple seasonal patterns (with annual, monthly, weekly and daily periodicities) and varying periods, due to the irregularity of the calendar.

Thirdly, in order to accommodate complex seasonal patterns, several harmonic cycles are needed to complement the fundamental one, or several season-specific individual effects are required. This poses variable selection or regularization problems of the kind that are typical of high-dimensional inferential settings. Particular attention must also be paid to modeling the effects of moving festivals, public holidays, and other calendar-induced irregularities.

Finally, robust estimation and filtering methods are crucial for addressing the high level of outlier contamination typically present in high-frequency data, due to limited aggregation and greater noise.

This course reviews key methodological solutions proposed in the literature and highlights open challenges that offer opportunities for further research. After reviewing the available approaches, it will focus on parametric and semiparametric models of seasonality within an unobserved components framework, where the seasonal component is estimated along with other components. Seasonality in high frequency data will be modelled from two main perspectives: the stochastic harmonic approach, based on the Fourier representation of a periodic function, and the time-domain random effects approach.

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Empirical applications in Python form an integral part of the course, with hands-on illustrations and software discussions.

2 Syllabus and Course Outline

1. Seasonality in economic, environmental and climatological time series.
2. Approaches to modeling seasonality: parametric, semi-parametric, and non-parametric methods. Deterministic vs. stochastic seasonality.
3. Regular stochastic seasonality models. Cycle models. Trigonometric specification. Time domain (dummy) specification; periodic splines.
4. Modelling multiple and non regular seasonal cycles. The stochastic harmonic approach. Time-domain random effect models; time-varying periodic splines.
5. Modelling holiday and calendar effects.
6. Statistical inference: estimation, model selection, and forecasting for complex seasonal time series. Robust filtering and forecasting techniques.
7. Empirical case studies in Python. Review of available software tools (e.g., STL, TBATS, Prophet).

3 References

Classic time series books that cover seasonality are Box et al. (2015), Harvey (1989) and Peña et al. (2001). Linton (2024) is a recent reference covering the topic.

General references for the analysis of regular seasonal economic time series are Hylleberg (1992), Ghysels and Osborn (2001), Proietti (2000), Bell et al. (2012), Proietti et al. (2016). Ghysels et al. (2006) deals with forecasting seasonal time series, providing some coverage of seasonality in financial time series.

For the Bayesian approach to modelling seasonal time series key references are West and Harrison (1997), Prado and West (2010) and Brodersen et al. (2015).

For the analysis of weekly and daily time series an essential list of readings includes Pierce et al. (1984), Harvey and Koopman (1993), Harvey et al. (1997), De Livera et al. (2011) and Proietti and Pedregal (2023). There is a large literature on the seasonality in electricity demand and price data, see for instance Koopman et al. (2007), and Weron (2014). For modelling and forecasting multiple seasonal components see also Gould et al. (2008), Taylor (2010), Kourentzes et al. (2014), and Xie and Ding (2020).

For parametric inference based on the Kalman filter and associated algorithms see Harvey (1989), Durbin and Koopman (2012). For the robust Kalman filter see Maronna et al. (2006), Marczak and Proietti (2016), Proietti and Pedregal (2023), among others, and for robust seasonal-trend decomposition using Loess (STL), see Wen et al. (2020).

Seasonal adjustment of high-frequency seasonal time series is discussed in Ollech (2023) and Webel and Smyk (2024).

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