Long-term climate forecast (in progress[2])

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Some Preliminaries

According to the IPCC, the study of climate change, and particularly global warming (GW), involves a careful analysis of the following four sequential issues:

1. What type of GW exists?
2. Causes of GW (is GW caused by human activities?)
3. Economic effects of GW, and
4. Economic policies to mitigate these effects.

This paper belongs to (1): What type of GW will exist in the future? And it is crucial for (3) and (4).

We answer this question by analyzing all the characteristics of the temperature distribution (as time series objects) and not only the average. **Think on how would YOU study the time evolution of the income distribution**.

Some Preliminaries

Why do we need to study Climate (or GW)?

There is a rapidly growing body of research (**Climate Economics**) analyzing the influence of climate change on economic outcomes. The extensive survey by Dell, Jones and Olken (JEL,2014) reviews the impact on:

- agricultural output, industrial output
- labor productivity, economic growth
- energy demand
- ***health***
- ***conflict***
- + ***sea level increase, extreme climate events (economic risk)***
- ++ mood, behavior, mental health, cognitive and psychological performance (see Taylor, *Frontiers in Physiology* 2015)
- +++ personality (Wenqui Wei and 25 co-authors in *Nature Human Behavior* 2017).

Some Preliminaries

How to Interpret $\underline{+x^oC}$ warmer than pre-industrial (1850-1900) levels?

The book ”Six Degrees: Our future on a hotter planet” by Mark Lynas (2008) can help:

- Last Ice Age (100000-10000 years ago) -4^oC with respect pre-industrial levels.
- $\underline{+1^oC}$ is NOW.
- $\underline{+2^oC}$ like 125000 years ago (Eemian interglacial period): North China thirsty, Peru glacier disappears, Ice loss would accelerate, Artic species threaten, sea level would increase, Oceans warmer and more acidic, etc.
- **TIPPING POINT** $\underline{+3^oC}$, three million years before (the Pliocene epoch): At that time CO2 360-400ppm (now higher than that), food problems, drought in Central America, Pakistan, western US or Australia, etc.
- $\underline{+4^oC}$, 40 million years ago, the Alps will resemble the Atlas of North Africa, Artic free of sea ice, an acceleration of the sea level rise (world's coasts under sentence of inundation).
- $\underline{+5^oC}$ like 55 million years deep into the past (Paleocene.Eocene Thermal Maximum), Alligators in the Artic, to be caught without shelter would be to die.
- $\underline{+6^oC}$ 144 to 65 millions years ago (the Cretaceous period): Sea levels 200 meters higher, ocean anoxia, 95% of all life was wiped out, ...

To warm up

The most popular graph showing the existence of GW is the *hockey stick* based on the annual MEAN temperature (across stations)

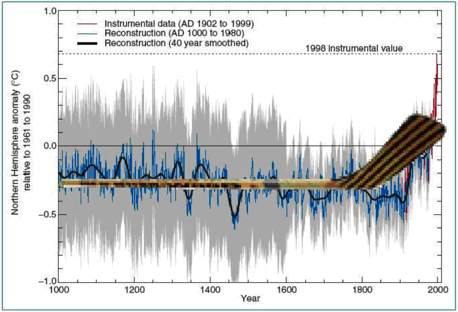
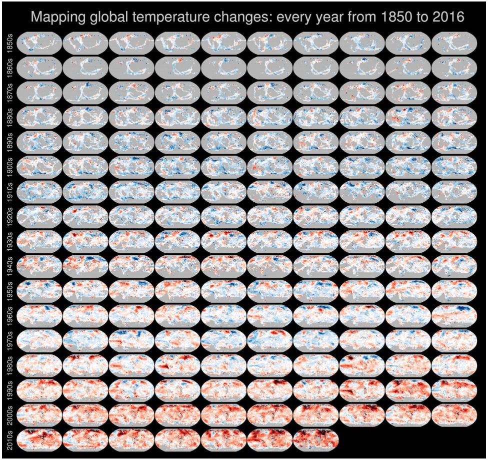


Figure 5: Millennial Northern Hemisphere (NH) temperature reconstruction (blue – tree rings, corals, ice cores, and historical records) and instrumental data (red) from AD 1000 to 1999. Smoother version of NH series (black), and two standard error limits (gray shaded) are shown. [Based on Figure 2.20]

It appears in the "Summary for Policy Makers" of the IPPC report 2001. It was constructed by Michael E. Mann and his colleagues. The term was coined by Jerry Mahlman (NOAA, Princeton)

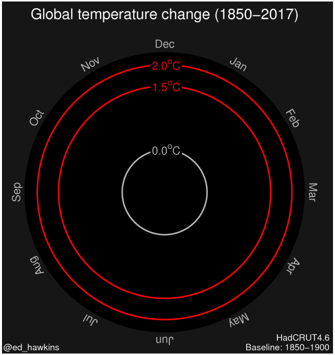
To warm up

How quickly climate change is **accelerating**, in 168 maps (Ed Hawkins, U. Reading):



To warm up

Global temperature change, spiral representation (Ed Hawkins):



http://www.climate-lab-book.ac.uk/spirals/

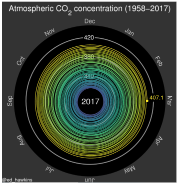
To warm up

Global temperature change, projections into the "future" 2100 (Jay Alder from the USGS):

https://www.usgs.gov/media/images/simulated-global-temperature-change-0

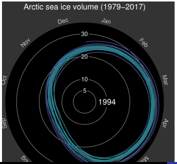
To warm up

Causes CO2:
http://www.climate-lab-book.ac.uk/files/2016/06/co2_2017.gif



co2

Ice in the Arctic:
http://www.climate-lab-book.ac.uk/files/2016/06/icevol_2017.gif



Ice

Introduction

- Climate is not a 7-14 days issue (this is weather).
- Climate is a Long-term issue. Therefore, Climate Forecasts must be long-term forecasts.
- These forecasts are key to design the right global warming mitigation economic policies.
- The **GOAL** of this paper is to produce long-term temperature forecasts. This will be done, by using the methodology introduced in Gadea and Gonzalo (*Trends in distributional characteristics: Existence of global warming*, JoE 2020) (GG). We will be able to produce Long-term temperature Density forecasts (*distributional characteristics*) that always convey more information than the standard mean forecasts shown in most of the literature.

Introduction

Some results:

- With respect to the pre-industrial period (1850-1900), the mean temperature will increase more than 2 degrees Celsius by 2040 and close to more than 4 degrees in one hundred years (2118). There will be similar absolute increases in the lower quantiles (q05 from 0C to 2C by 2040 and to 4C by 2118) and a smaller increase in the upper quantiles (q95 from 25.5C to 27C by 2040 and to 27.6C by 2118).
- The interquartile range (similar results with the variance) of the density forecast keeps decreasing (*Great Compression*) according to the long-run forecasts. Similar shrinking in-sample results have been found in GG for the whole Globe and for Central England and, recently, Diebold and Rudebusch (2019) for the US.
- All the previous results are obtained using the whole sample 1880-2018. When the models are re-estimated with more recent samples, there is a clear acceleration process.

Introduction

The long-term predictions are obtained by following the next steps:

1. By GG we know there exists a trend in all the characteristics of the temperature distribution; but we do not know which type of trend. So we start by proposing a set of families of trend models: $M_1, M_2, ..., M_m$.
2. In each model family, for instance $M_2 =$ (*polynomial trend*) we select in-sample the best model M_2^* via an information criteria (BIC, AIC, ...).
3. By using the direct forecasting method (predictive regressions), we construct mean (q05, q50, q95,...) forecasts for h (1, 10, 25, 50, 100) years ahead. This method consists of regressing the mean (q05, q50, q95, ...) time series at time "t+h" on the model information at time "t". The forecast is the estimated latter part.
4. For a given h, we run an out of sample forecasting competition among all the mean (q05, q50, q95, ...) models and select the Pareto superior models via a forecasting evaluation test.
5. To eliminate the forecast model dependency (uncertainty), we propose FORECAST COMBINATIONS (model averaging, simple average, Bates and Granger 1969, etc.): (i) All the models (some trimming) and (ii) only Pareto superior models.

Outline

GG 2020 Methodology

- For the purpose of this research, global temperatures are seen as a stochastic process, $X = (X_t(\omega), t \in T)$, where T is an interval in $\Re$, defined on a probability space $(\Omega, \Im, P)$ and such that $t \to X_t(\omega)$ belongs to some function space **G** for all $\omega \in \Omega$. X defines a **G-valued stochastic process**. This function space **G** is equipped with a scalar product $< ., . >$, and or a norm $\| \ . \ \|$ and a *Borel* σ *-algebra*, B_G. It is separable and complete.
- **G** can be a Hilbert space as in Bosq (2000) (AR-H model for sequences of random Hilbert functions $X_1(\omega), X_2(\omega), ..., X_T(\omega)$), as in Chang, Park and co-autors (2012, 2015) (regression models for sequences of random state densities $f_1(\omega), f_2(\omega), ..., f_T(\omega)$) or a Banach space for sequence of random state distributions $(F_1(\omega), F_2(\omega), ..., F_T(\omega))$, etc.
- A convenient EXAMPLE of an infinite-dimensional discrete-time process consists on associating with $\xi = (\xi_n, n \in \Re_+)$ a sequence of random variables with values in an appropriated function space. This may be obtained by setting

$$X_t(n) = \xi_{tN+n}, 0 \leq n \leq N, t = 0, 1, 2, ..., T,$$

so $X = (X_t, t = 0, 1, 2, ..., T)$. If the sample paths of ξ are continuous, then we have a sequence $X_0, X_1, ...$ of random variables in $C[0, N]$.

GG 2020 Methodology

Using a distributional characteristic approach, we move from Ω to $\mathbb{R}^T$, as in a standard stochastic process, passing through a **G** functional space:

$$\underset{(\omega)}{\Omega} \xrightarrow{X} \underset{X_t(\omega)}{\mathbf{G}} \xrightarrow{C} \underset{C_t(\omega)}{\mathbb{R}}$$

Returning to the convenient example and abusing the notation, the stochastic structure can be summarized in the following array:

$X_{10}(\omega) = \xi_0(\omega)$	$X_{11}(\omega) = \xi_1(\omega)$	$\dots$	$X_{1N}(\omega) = \xi_N(\omega)$	$C_1(\omega)$
$X_{20}(\omega) = \xi_{N+1}(\omega)$	$X_{21}(\omega) = \xi_{N+2}(\omega)$	$\dots$	$X_{2N}(\omega) = \xi_{2N}(\omega)$	$C_2(\omega)$
$\vdots$	$\vdots$	$\dots$	$\vdots$	$\vdots$
$X_{T0}(\omega) = \xi_{(T-1)N+1}(\omega)$	$X_{T1}(\omega) = \xi_{(T-1)N+2}(\omega)$	$\dots$	$X_{TN}(\omega) = \xi_{TN}(\omega)$	$C_T(\omega)$

GG method

A first look at the data

Let's go to the data!!!!!! (...for a second)
Remember: Climate is the statistics of weather
Two types of data:

1. Cross-sectional dimension per year.
   Global temperature across stations from 1850 to 2019. (CRU)
2. Time-series dimension per year.
   We use thermometer recorded daily temperature data at Central England, from January 1st 1772 to June 31st 2019. (we discard observations of 2019) (CET)

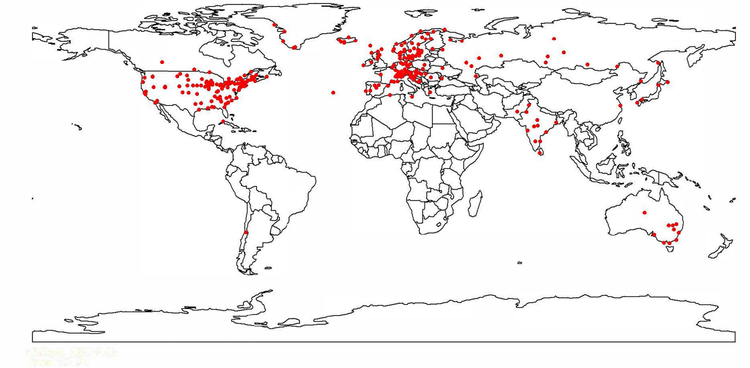
A first look at the data

We extract several characteristics of the temperature distribution as time series objects:

1. mean
2. maximum
3. minimum
4. standard deviation
5. inter-quartile range
6. range
7. kurtosis
8. skewness
9. q05
10. q10
11. q20
12. q30
13. q40
14. q50
15. q60
16. q70
17. q80
18. q90
19. q95

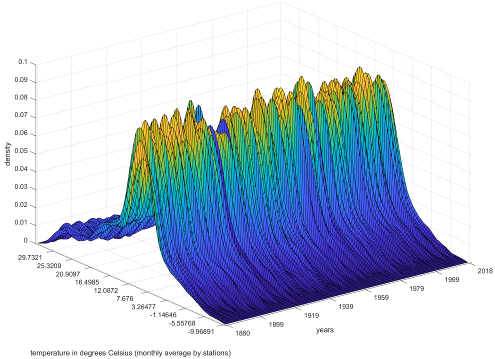
A first look at the data

Cross-sectional data (The Whole Globe)



A first look at the data

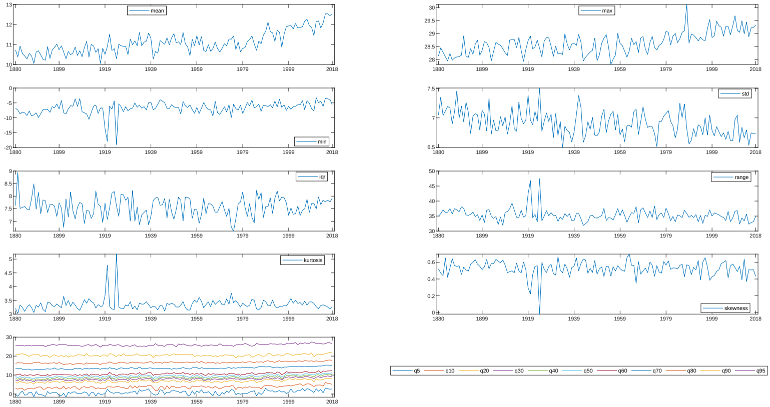
Cross-sectional data (The Whole Globe)



In 2019 more than 10,000 stations. We select stations that provide temperature data for every month from 1880 to 2018: N= 157.

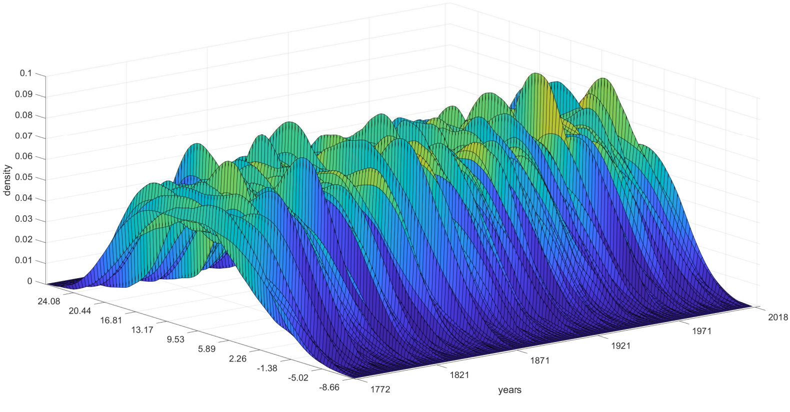
A first look at the data

Cross-secional data (The Whole Globe)



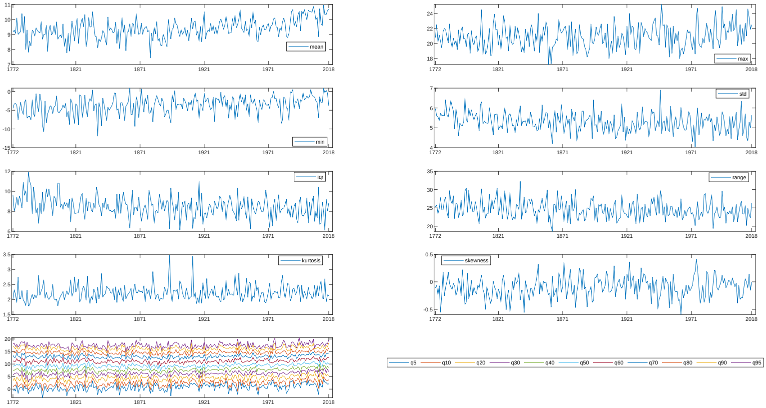
A first look at the data

Time-series data (Central-England)



A first look at the data

Time-series data (Central-England)



Definition of Global Warming

Definition of Glogal Warming

Global Warming is defined as the existence of an increasing trend in some of the characteristics measuring the central tendency or position (quantiles) of the global temperature distribution.

What is a trend?
We follow the intuitive definition given in White and Granger (2011) (WG): (i) a trend should have a direction; (ii) a trend should be basically smooth; (iii) a trend does not have to be monotonic throughout; and (iv) a trend can be a local behavior (observed trends can be related to a particular section of data). These characterizations are formalized by WG in the following two definitions, one for deterministic trends and the other for stochastic trends.

What is a trend?

Definition 1: Deterministic trend (WG, 2011)

Let $\{C_t\} = \{C_t : t = 0, 1, ...\}$ be a sequence of real numbers. If $C_t \leq C_{t+1}$ for all t, then $\{C_t\}$ is an increasing (weak) trend. If $\{-C_t\}$ is an increasing (weak) trend, then $\{C_t\}$ is a decreasing (weak) trend.

Definition 2: Stochastic trend (WG, 2011)

Let X_t be a stochastic process.

- Let $C_t = E(X_t)$. If C_t is a (weak) increasing trend, then $\{X_t\}$ has a (weak) increasing trend in mean.
- Let $C_t = E(|X_t - E(X_t)|^k)$ for finite positive real k. If C_t is a (weak) increasing trend, then $\{X_t\}$ has a (weak) increasing trend in k^{th} absolute central moment.
- Let $C_t(p) = inf\{x \epsilon \Re : F_t(x) \geq p\}$ be the quantile $p \in (0, 1)$ of the distribution function $F_t(x) = P(X_t \leq x)$. If $C_t(p)$ is a (weak) increasing trend, then $\{X_t\}$ has a (weak) increasing trend in quantile p.

A Practical definition

Definition 3: Practical definition of trend (GG 2020)

A characteristic C_t of a stochastic process Y_t contains a trend if $\beta \neq 0$ in the regression

$$C_t = \alpha + \beta t + u_t. \tag{1}$$

▸ Proposition 1
▸ Proposition 2
▸ Proposition 3

Cross-sectional data: Unit roots

Table 1: Unit root tests (CRU data)

Characteristic	ADF-SBIC	p-value	lags
mean	-4.65	0.002	1
max	-8.57	0.000	0
min	-10.85	0.000	0
std	-11.10	0.000	0
iqr	-7.62	0.000	1
rank	-10.77	0.000	0
kur	-4.28	0.005	3
skw	-11.52	0.000	0
q05	-10.29	0.000	0
q10	-8.45	0.000	0
q20	-5.67	0.000	1
q30	-8.69	0.000	0
q40	-5.14	0.000	1
q50	-5.35	0.000	1
q60	-5.59	0.000	1
q70	-2.40	0.394	3
q80	-5.03	0.000	1
q90	-8.23	0.000	0
q95	-7.60	0.000	0

Cross-sectional data: Testing for trends

Table 2: Linear trend with OLS-HAC (CRU data)

Characteristic	Coeff	p-value
mean	0.0107	0.0000
max	0.0069	0.0000
min	0.0188	0.0000
std	-0.0023	0.0000
iqr	-0.0000	0.4817
rank	-0.0119	0.0024
kur	0.0003	0.2839
skw	-0.0001	0.2473
q05	0.0127	0.0000
q10	0.0122	0.0000
q20	0.0112	0.0000
q30	0.0119	0.0000
q40	0.0122	0.0000
q50	0.0118	0.0000
q60	0.0114	0.0000
q70	0.0116	0.0000
q80	0.0097	0.0000
q90	0.0035	0.0247
q95	0.0083	0.0000

Competing models

SO THERE IS A TREND; BUT WHAT TYPE OF TREND???

1. Unconditional mean model (mean)
2. Linear-trend model (linear-trend)
3. Polynomial trend model of degree k (pol-trend)
4. Slope-average polynomial trend model (pol-trend-av-sl) av-sl model
5. Logarithmic polynomial trend model of degree k, proxy for a slowly varying trend model (log-trend)
6. Polynomial trend autoregressive model of degree k and order p (pol-trend-arp)
7. Slope-average polynomial trend autoregressive model of degree k and order p (pol-trend-arp-av-sl)
8. Autoregressive model of order p (arp)
9. Random walk model (rw)
10. Random walk + drift model (rwd)
11. IMA model (ima)
12. ARFIMA model (0,d,0) (arfima)
13. AR model with p=20, proxy for an arfima model (arp20)

Outline

Roadmap for the MEAN forecast

- The MEAN temperature time series is a time series object (it could have been q05 or any other temperature distributional characteristic).
- Inside each model family, we select the best (in-sample) model according to BIC. With each ”best ones” of theses models we forecast out-sample ($h = 1, 10, 25, 50, 100$, *etc.*) by using the DIRECT method. Forecast confidence intervals are constructed by standard methods as well as by Quantile Regressions (QR) (see Adrian et al., AER 2019).
- For each horizon, select the "Pareto" forecasting superior models via an out-sample forecasting competition, using Giacomini and White (Ecta 2006) rolling window test (GW). GW test is used to avoid the nested model problem.
- Be aware!!! Some in-sample non-linear trend models do "weird" things out-sample in the long-run.
- Forecast combinations: All the models, some trimming, only the "Pareto" forecasting superior models, etc. Confidence models as an alternative to select a set of models in the forecast competition (Hansen et al., Ecta 2001).
- THINK about what type of temperature information do we obtain when we produce point forecasts of the Mean time series object together with its confidence intervals. Are we saying much about the future temperature distribution? Ummmmmm ...

Mean temperature Model Selection

Table 3: BIC model selection for the mean temperature (CRU data, 1880-2018)

Models	BIC	weights
mean	-1.1742	0.0548
linear-trend	-1.9988	0.0827
pol-trend (k=5)	-2.2521	0.0939
pol-trend-av-sl (k=5)	-2.2521	0.0939
pol-trend-log (k=8)	-2.1522	0.0893
pol-trend-arp (k=3,p=1)	-2.2516	0.0938
pol-trend-arp-av-sl(k=3,p=1)	-2.2516	0.0858
arp (p=2)	-2.0729	0.0858
rw	-1.8930	0.0784
rwd	-1.8646	0.0773
ima	-2.2366	0.0931
arfima	-2.1144	0.0876
arp20	-1.6474	0.0694

Note. Weights are calculated as follows: $w_j = \frac{e^{-\frac{1}{2}BIC_j}}{\sum_{j=1}^{m} e^{-\frac{1}{2}BIC_j}}$ where BIC_j is the BIC criterium of each model from j=1...m in sample.

Mean temperature forecast with BIC-selected model

Table 4: Mean forecast with *pol-trend* model ($k = 5$) (CRU data, 1880-2018)

Horizon	LS point forecast	CI	LAD point forecast	CI
1	12.46	(11.97,12.95)	12.53	(12.27,12.63)
10	12.68	(12.19,13.17)	12.85	(13.72,12.65)
25	12.29	(11.80,12.78)	12.84	(18.34,11.50)
50	6.57??	(6.08,7.06)	8.87??	(37.13,2.05)
100	-60.55??	(-61.04,-60.06)	-44.41??	(172.58,-96.60)

Note. The average value of the Mean in the full sample is 11.09; the average value of the Mean in 1880-1900 (the baseline model for the Paris Agreement) is 10.57; the average value of the Mean in the climatologist reference period (1986-2005) is 11.63; the last value of the sample, in 2018, is 12.54.

Mean temperature forecast with BIC-selected model

Table 5: Mean forecast with *pol-trend-av-sl* model (CRU data, 1880-2018)

Horizon	LS point forecast	CI	LAD point forecast	CI
1	12.47	(11.98,12.96)	12.54	(12.19,12.65)
10	12.59	(12.10,13.08)	12.66	(12.36,12.75)
25	12.78	(12.29,13.27)	12.86	(12.65,12.93)
50	13.10	(12.61,13.59)	13.20	(13.12,13.22)
100	13.73	(13.24,14.22)	13.87	(14.07,13.80)

Note. The average value of the Mean in the full sample is 11.09; the average value of the Mean in 1880-1900 (the baseline model for the Paris Agreement) is 10.57; the average value of the Mean in the climatologist reference period (1986-2005) is 11.63; the last value of the sample, in 2018, is 12.54.

Mean temperature forecast with benchmark model: linear trend

Table 6: Mean forecast with benchmark model, *linear trend* model (CRU data, 1880-2018)

Horizon	LS point forecast	CI	LAD point forecast	CI
1	11.83	(11.24,12.42)	11.89	(10.98,12.56)
10	11.93	(11.34,12.52)	11.99	(11.05,12.67)
25	12.09	(11.50,12.68)	12.15	(11.15,12.86)
50	12.36	(11.77,12.94)	12.42	(11.33,13.18)
100	12.89	(12.30,13.48)	12.96	(11.69,13.81)

Note. The average value of the Mean in the full sample is 11.09; the average value of the Mean in 1880-1900 (the baseline model for the Paris Agreement) is 10.57; the average value of the Mean in the climatologist reference period (1986-2005) is 11.63; the last value of the sample, in 2018, is 12.54.

Acceleration hypothesis for the Mean

Table 7: Acceleration hypothesis: Mean temperature forecast with benchmark model, *linear-trend* model using the last 70 years (CRU data, 1949-2018)

Horizon	LS point forecast	CI	LAD point forecast	CI
1	12.13	(11.55,12.71)	12.17	(11.33,12.58)
10	12.31	(11.73,12.89)	12.36	(11.46,12.71)
25	12.62	(12.04,13.20)	12.67	(11.67,12.94)
50	13.13	(12.55,13.71)	13.19	(12.03,13.31)
100	14.15	(13.57,14.73)	14.22	(12.74,14.05)

Note. The average value of the Mean in the full sample is 11.09; the average value of the Mean in 1880-1900 (the baseline model for the Paris Agreement) is 10.57; the average value of the Mean in the climatologist reference period (1986-2005) is 11.63; the last value of the sample, in 2018, is 12.54. The average value of the Mean in the last 70 years is 11.40.

Remarks

Some Remarks:

- With respect to the $2^o C$ increase of **the Paris agreement** we find that in 10-25 years the mean temperature will increase $\sim 2^0 C$. In 100 years the mean temperature will increase $\geq 3^0 C$. *These numbers are very similar to the ones obtained from climate models projections under the RCP 8.5 scenario of "business as usual"*.
- There is a long-term acceleration: for $h = 100$ the increment in the forecast with a linear trend model almost DOUBLE (x1.5).
- Be careful with long-run out-sample forecasts using in-sample non-linear models.
- Forecasts are model dependent (model choice may be critical). Try Forecast Combination.
- Think about what are we forecasting when we use the mean temperature (and its confidence intervals).

Mean temperature forecast combination

Table 8: Long-term forecast for the mean (CRU data, 1880-2018)

Models/horizon	h=1	h=25	h=100
	mean	mean	mean
mean	11.09	11.09	11.09
linear-trend	11.83	12.09	12.89
pol-trend	12.46	12.29	-60.55
pol-trend-av-sl	12.47	12.78	13.73
pol-trend-log	12.67	15.10	33.36
pol-trend-arp	12.53	14.85	131.80
pol-trend-arp-av-sl	12.51	13.01	98.61
arp	12.33	12.34	13.54
rw	12.54	12.54	12.54
rwd	12.56	12.87	13.86
ima	12.40	12.72	13.70
arfima	12.26	11.62	11.38
arp20	12.41	12.54	13.49
combined	12.35	12.83	25.88
combined*	12.39	12.72	23.86
combined**	12.44	12.58	15.52

Note. The average value of the Mean in the full sample is 11.09; the average value of the Mean in 1880-1900 (the baseline model for the Paris Agreement) is 10.57; the average value of the Mean in the climatologist reference period (1986-2005) is 11.63; the last value of the sample, in 2018, is 12.54. “Combination” weights the models using the BICs obtained in-sample; “combination*” removes two extreme values and “combination**” four extreme values. In these last cases BIC-weights are properly recalculated.

Mean temperature forecast combination

Table 9: Long-term density forecast for the *mean* with "Pareto" superior models (CRU data, 1880-2018)

Models/horizon	h=1	h=10	h=25
	mean	mean	mean
mean	-	-	-
linear-trend	-	-	12.09
pol-trend	12.46	-	-
pol-trend-av-sl	12.47	12.59	12.78
pol-trend-log	12.67	-	-
pol-trend-arp	12.53	-	-
pol-trend-arp-av-sl	-	-	-
arp	-	-	-
rw	12.54	-	-
rwd	12.56	-	-
ima	12.40	-	-
arfima	-	-	-
arp20	-	-	-
comb1	12.52	12.59	12.46
comb2	12.52	12.59	12.43
comb3	12.46	12.59	12.44

Note. comb1: BIC weights; comb2: sample average; comb3: $\beta' s$ from minimizing MSEs, following Bates and Granger (1969).

The average value of the Mean in the full sample is 11.09; the average value of the Mean in 1880-1900 (the baseline model for the Paris Agreement) is 10.57; the average value of the Mean in the climatologist reference period (1986-2005) is 11.63; the last value of the sample, in

Mean temperature forecast combination

Table 10: Temperature forecast combinations with "Pareto" superior models for the mean (CRU data, 1880-2018)

	q05					
	comb1		comb2		comb3	
forecast horizon	Forecast	CI	Forecast	CI	Forecast	CI
h=1	12.52	(11.98,13.05)	12.52	(11.98,13.06)	12.46	(11.90,13.03)
h=10	12.59	(12.10,13.08)	12.59	(12.10,13.08)	12.59	(12.10,13.08)
h=25	12.46	(11.92,12.99)	12.43	(11.89,12.97)	12.44	(11.91,12.98)

Note. comb1: BIC weights; comb2: sample average; comb3: $\beta's$ from minimizing MSEs, following Bates and Granger (1969).

The average value of the Mean in the full sample is 11.09; the average value of the Mean in 1880-1900 (the baseline model for the Paris Agreement) is 10.57; the average value of the Mean in the climatologist reference period (1986-2005) is 11.63; the last value of the sample, in 2018, is 12.54.

Mean temperature forecast combination (all the models)

Table 11: Mean temperature forecast combining all models (CRU data, 1880-2018)

	comb1		comb2		comb3	
horizon	Forecast	CI	Forecast	CI	Forecast	CI
h=1	12.35	(11.77,12.93)	12.31	(11.72,12.91)	12.15	(11.55,12.75)
h=10	12.54	(11.94,13.13)	12.49	(11.88,13.10)	12.09	(11.51,12.67)
h=25	12.83	(12.21,13.45)	12.76	(12.12,13.39)	12.30	(11.62,12.98)

Note. comb1: BIC weights; comb2: sample average; comb3: $\beta' s$ from minimizing MSEs, following Bates and Granger (1969).

The average value of the Mean in the full sample is 11.09; the average value of the Mean in 1880-1900 (the baseline model for the Paris Agreement) is 10.57; the average value of the Mean in the climatologist reference period (1986-2005) is 11.63; the last value of the sample, in 2018, is 12.54.

Remarks

Some Remarks:

- Long-run forecast in trend models are model dependent (uncertainty).
- Forecast combination could eliminate this dependence; but there are many ways to combine forecasts.
- Sometimes forecast combination "smooth" the forecasts. We still find that with respect to the $2^o C$ increase of the Paris agreement in 10-25 years the mean temperature will increase $\sim 2^0 C$.
- How to combine forecasts? This is now a "hot" topic again in AI, Machine Learning, Big Data, etc.
- Our proposal in this paper is to combine ”Pareto” superior models according to some out of sample evaluation. BUT if you want to use ALL the models, then use some trimming combination.

Outline

Remarks

Some Remarks:

- In our methodology, q05 is a time series object like the standard Mean.
- q05 represents a "region" in the globe (CRU data) or a "season" in a particular part of the globe (CET data).
- We can repeat the same forecast exercise done for the Mean with q05. It is a way of capturing certain part of the existing climate heterogeneity.
- Notice that the q05 forecast results can NOT be inferred from the Mean forecast analysis.
- We clearly need a simple Density forecast approach if we want to capture the whole climate heterogeneity (the Artic is not the same as California).

Model Selection for q05 temperature

Table 12: BIC model selection for the q05 temperature (CRU data, 1880-2018)

Models	BIC	weights
mean	0.1051	0.0820
linear-trend	-0.0905	0.0905
pol-trend (k=3)	-0.0967	0.0908
pol-trend-av-sl (k=3)	-0.0967	0.0908
pol-trend-log (k=2)	-0.0336	0.0879
pol-trend-arp (k=1,p=1)	-0.0612	0.0892
pol-trend-arp-av-sl (k=1,p=1)	-0.0508	0.0834
arp (p=2)	0.0527	0.0842
rw	0.4183	0.0702
rwd	0.4467	0.0692
ima	-0.0871	0.0903
arfima	-0.0092	0.0869
arp20	0.4761	0.0682

Note. Weights are calculated as follows: $w_j = \frac{e^{-1/2BIC_j}}{\sum_{j=1}^{m} e^{-1/2BIC_j}}$ where BIC_j is the BIC criterium of each model from j=1...m in sample.

q05 temperature forecast with BIC-selected model

Table 13: q05 temperature forecast with BIC-selected model, *pol-trend* model ($k = 3$) (CRU data, 1880-2018)

Horizon	LS point forecast	CI	LAD point forecast	CI
1	2.60	(1.12,4.08)	2.79	(0.90,3.27)
10	3.37	(1.89,4.85)	3.71	(1.51,3.62)
25	5.08	(3.60,6.56)	5.78	(2.84,4.37)
50	9.35??	(7.87,10.83)	11.01??	(6.11,6.18)
100	24.73??	(23.25,26.21)	30.19??	(17.67,12.57)

Note. The average value of the q05 in the full sample is 0.850; the average value of q05 in 1880-1900 (the baseline model for the Paris Agreement) is 0.07; the average value of q05 in the climatologist reference period (1986-2005) is 1.45; the last value of the sample, in 2018, is 2.70.

q05 temperature forecast with BIC-selected model

Table 14: q05 temperature forecast with BIC-selected model, *pol-trend-av-sl* model ($k = 3$) (CRU data, 1880-2018)

Horizon	LS point forecast	CI	LAD point forecast	CI
1	2.55	(1.07,4.03)	2.73	(0.87,3.26)
10	2.75	(1.27,4.23)	2.96	(1.04,3.37)
25	3.07	(1.59,4.55)	3.34	(1.33,3.56)
50	3.61	(2.13,5.09)	3.98	(1.82,3.88)
100	4.69	(3.21,6.18)	5.26	(2.79,4.51)

Note. The average value of the q05 in the full sample is 0.850; the average value of q05 in 1880-1900 (the baseline model for the Paris Agreement) is 0.07; the average value of q05 in the climatologist reference period (1986-2005) is 1.45; the last value of the sample, in 2018, is 2.70.

q05 temperature forecast with benchmark model: linear trend

Table 15: q05 temperature forecast with benchmark model, *linear-trend* model (CRU data, 1880-2018)

Horizon	LS point forecast	CI	LAD point forecast	CI
1	1.74	(0.21,3.27)	1.76	(0.13,3.05)
10	1.86	(0.33,3.39)	1.87	(0.24,3.15)
25	2.05	(0.52,3.58)	2.05	(0.43,3.31)
50	2.37	(0.84,3.89)	2.35	(0.74,3.58)
100	3.00	(1.47,4.53)	2.96	(1.37,4.12)

Note. The average value of the q05 in the full sample is 0.850; the average value of q05 in 1880-1900 (the baseline model for the Paris Agreement) is 0.07; the average value of q05 in the climatologist reference period (1986-2005) is 1.45; the last value of the sample, in 2018, is 2.70.

Acceleration hypothesis for q05 temperature

Table 16: Acceleration hypothesis: q05 temperature forecast with BIC-selected model, *pol-trend* (K=2)model using the last 70 years (CRU data, 1949-2018)

Horizon	LS point forecast	CI	LAD point forecast	CI
1	2.78	(1.33,4.23)	2.94	(0.99,3.19)
10	3.63	(2.18,5.07)	3.89	(1.78,3.29)
25	5.35	(3.91,6.80)	5.84	(3.39,3.42)
50	9.10??	(7.66,10.55)	10.15??	(6.90,3.50)
100	19.90??	(18.45,21.34)	22.66??	(17.04,3.19)

Note. The average value of the q05 in the full sample is 0.850; the average value of q05 in 1880-1900 (the baseline model for the Paris Agreement) is 0.07; the average value of q05 in the climatologist reference period (1986-2005) is 1.45; the last value of the sample, in 2018, is 2.70. The average value of q05 in the last 70 years is 1.19.

Acceleration hypothesis for q05 temperature

Table 17: Acceleration hypothesis: q05 temperature forecast with BIC-selected model, *pol-trend-av-sl* model with the 70 last years (CRU data, 1949-2018)

Horizon	LS point forecast	CI	LAD point forecast	CI
1	2.78	(1.33,4.23)	2.94	(0.99,3.19)
10	3.00	(1.55,4.44)	3.13	(1.18,3.38)
25	3.35	(1.90,4.80)	3.47	(1.49,3.70)
50	3.95	(2.50,5.39)	4.02	(2.02,4.22)
100	5.13	(3.69,6.58)	5.12	(3.06,5.27)

Note. The average value of the q05 in the full sample is 0.850; the average value of q05 in 1880-1900 (the baseline model for the Paris Agreement) is 0.07; the average value of q05 in the climatologist reference period (1986-2005) is 1.45; the last value of the sample, in 2018, is 2.70. The average value of q05 in the last 70 years is 1.19.

Acceleration hypothesis for q05 temperature

Table 18: Acceleration hypothesis: q05 temperature forecast with benchmark model, *linear-trend* model with the 70 last years (CRU data 1949-2018)

Horizon	LS point forecast	CI	LAD point forecast	CI
1	2.03	(0.50,3.57)	1.91	(0.77,3.22)
10	2.25	(0.72,3.78)	2.11	(1.06,3.38)
25	2.60	(1.07,4.14)	2.44	(1.53,3.65)
50	3.20	(1.67,4.73)	2.99	(2.32,4.11)
100	4.39	(2.85,5.92)	4.09	(3.91,5.03)

Note. The average value of the q05 in the full sample is 0.850; the average value of q05 in 1880-1900 (the baseline model for the Paris Agreement) is 0.07; the average value of q05 in the climatologist reference period (1986-2005) is 1.45; the last value of the sample, in 2018, is 2.70. The average value of q05 in the last 70 years is 1.19.

Remarks

Some Remarks:

- In 100 years the q05 temperature will increase more than the Mean: amplification effect.
- In 100 years the q05 temperature will increase with some models more than 3.4C degrees (with respect the pre-industrial period, 1850-1900).
- There is a long-term acceleration. It is stronger than the one for the Mean.
- Be careful with long-run out-sample forecasts using in-sample non-linear models.
- Forecasts are model dependent (model choice may be critical). Try Forecast Combination.
- Think if from the Mean forecast exercise you could have inferred the q05 results.

q05 temperature forecast combination with all models

Table 19: q05 temperature forecast combining all models (CRU data, 1880-2018

	comb1		comb2		comb3	
horizon	Forecast	CI	Forecast	CI	Forecast	CI
h=1	1.99	(0.33,3.65)	2.00	(0.33,3.67)	1.60	(-0.03,3.22)
h=10	2.16	(0.51,3.82)	2.18	(0.51,3.85)	2.31	(0.81,3.81)
h=25	2.23	(0.55,3.90)	2.21	(0.52,3.90)	2.22	(0.69,3.74)

Note. comb1: BIC weights; comb2: sample average; comb3: $\beta's$ from minimizing MSEs, following Bates and Granger (1969).

The average value of the q05 in the full sample is 0.850; the average value of q05 in 1880-1900 (the baseline model for the Paris Agreement) is 0.07; the average value of q05 in the climatologist reference period (1986-2005) is 1.45; the last value of the sample, in 2018, is 2.70.

q05 temperature forecast combination with "Pareto" superior models

Table 20: q05 temperature forecast combination with “Pareto” superior models (CRU data, 1880-2018)

	comb1		comb2		comb3	
horizon	Forecast	CI	Forecast	CI	Forecast	CI
h=1	2.09	(0.49,3.70)	2.09	(0.48,3.70)	2.15	(0.29,4.01)
h=10	2.43	(0.94,3.92)	2.43	(0.94,3.92)	2.29	(0.80,3.79)
h=25	2.43	(0.89,3.97)	2.43	(0.89,3.97)	2.10	(0.57,3.63)

Note. comb1: BIC weights; comb2: sample average; comb3: $\beta's$ from minimizing MSEs, following Bates and Granger (1969).

The average value of the q05 in the full sample is 0.850; the average value of q05 in 1880-1900 (the baseline model for the Paris Agreement) is 0.07; the average value of q05 in the climatologist reference period (1986-2005) is 1.45; the last value of the sample, in 2018, is 2.70.

Remarks

Some Remarks:

- We get similar lessons that in the case of the mean
  1. Forecasts are model dependent.
  2. Forecast combinations could eliminate this dependence; but there are many ways to combine forecasts.
  3. Sometimes forecast combination "smooth" the forecasts.
  4. The question is how to combine forecasts?
- Until now we have obtained forecasts for the mean and the q05 quantile.

Outline

Remarks

Some Remarks:

- The Mean as well as the q05 forecasts offer an incomplete forecasting picture.
- With our methodology we will repeat the same forecasting exercise for q05, q50 and q95 (and any other quantile we may need).
- (I) The best models of each trend model family are used to produced forecasts for each of the three quantiles and different horizons. Trimming average forecast combinations are produced.
- (II) We run an out-sample forecast competition for each of the three quantiles and different horizons. Once the "Pareto" superior models are obtained, their forecast will be combined for each h.
- (III) (still on progress) A multivariate version of (II): the same model for all the quantiles.

Long-term temperature Density forecast

Table 21: Long-term temperature density forecast (CRU data, 1880-2018)

Models/horizon	h=1			h=25			h=100		
	q5	q50	q95	q5	q50	q95	q5	q50	q95
mean	0.85	9.48	25.87	0.85	9.48	25.87	0.85	9.48	25.87
linear-trend	1.74	10.30	26.45	2.05	10.58	26.65	3.00	11.47	27.27
pol-trend	2.60	11.06	26.91	5.08	13.11	27.66	24.73	28.51	31.03
pol-trend-av-sl	2.55	11.03	26.91	3.07	11.47	27.11	4.69	12.84	27.73
pol-trend-log	1.55	10.98	26.96	1.71	12.45	27.72	2.12	19.79	30.66
pol-trend-arp	1.80	11.14	26.94	1.92	13.62	27.81	6.86	182.55	14.36
pol-trend-arp-av-sl	1.80	11.11	26.94	1.92	11.55	27.18	6.86	130.74	40.30
arp	1.65	10.80	26.64	1.43	10.48	26.88	1.70	12.03	26.82
rw	2.70	11.36	26.50	2.70	11.36	26.50	2.70	11.36	26.50
rwd	2.72	11.37	26.51	3.23	11.73	26.67	4.83	12.85	27.19
ima	2.30	10.93	26.75	2.82	11.29	26.92	4.42	12.41	27.43
arfima	1.82	10.76	26.52	1.08	9.94	26.19	0.93	9.69	26.03
arp20	1.91	10.75	26.97	0.81	10.78	28.39	6.50	12.22	29.57
combined	1.99	10.88	26.70	2.23	11.45	27.05	5.52	38.56	27.78
combined*	2.03	10.93	26.74	2.05	11.37	27.05	4.03	26.17	27.88
combined**	2.03	10.96	26.75	2.07	11.32	27.05	4.06	15.12	27.66

Note. The average values of q05, q50 and q95 for the full sample are, respectively, 0.85, 9.48 and 25.87; in the Paris agreement baseline period (1880-1900) are 0.07, 8.84 and 25.64; in the climatologists reference period (1986-2005) are 1.45, 10.05 and 26.38. Values at the end of the sample are: 2.70, 11.36 and 26.50. "Combination" weights the models using the BICs obtained in-sample; "combination*" removes two extreme values and "combination**" four extreme values. In these last cases BIC-weights are properly recalculated.

Long-term temperature Density forecast

Table 22: Long-term density forecast for $q05$, $q50$ and $q95$ with "Pareto" superior models (CRU data, 1880-2018)

Models/horizon	h=1			h=10			h=25		
	q05	q50	q95	q5	q50	q95	q05	q50	q95
mean	-	-	-	-	-	-	-	-	-
linear-trend	1.74	-	-	-	-	-	2.05	10.58	-
pol-trend	2.60	11.06	26.91	-	11.71	27.17	-	13.11	27.66
pol-trend-av-sl	2.55	11.03	26.91	2.75	11.20	-	-	-	-
pol-trend-log	-	10.98	26.96	-	11.48	27.23	-	-	27.72
pol-trend-arp	1.80	11.14	26.94	-	-	27.23	-	-	27.81
pol-trend-arp-av-sl	1.80	-	-	2.11	-	-	-	11.55	27.18
arp	-	-	-	-	-	-	-	-	-
rw	-	11.36	26.50	-	-	-	-	-	-
rwd	-	11.37	26.51	-	11.51	-	-	11.73	-
ima	2.30	10.93	26.75	-	11.07	-	2.82	11.29	-
arfima	1.82	10.76	26.52	-	-	-	-	-	-
arp20	-	10.75	-	-	-	-	-	-	-
comb1	2.09	11.04	26.76	2.43	11.39	27.21	2.43	11.67	27.59
comb2	2.09	11.04	26.75	2.43	11.39	27.21	2.43	11.65	27.59
comb3	2.15	11.05	26.66	2.29	11.09	27.17	2.10	11.37	27.72

Note. comb1: BIC weights; comb2: sample average; comb3: $\beta' s$ from minimizing MSEs, following Bates and Granger (1969). The average values of q05, q50 and q95 for the full sample are, respectively, 0.85, 9.48 and 25.87; in the Paris agreement baseline period (1880-1900) are 0.07, 8.84 and 25.64; in the climatologists reference period (1986-2005) are 1.45, 10.05 and 26.38. Values at the end of the sample are: 2.70,

Long-term temperature Density forecast

Table 23: Temperature forecast combinations with "Pareto" superior models for *q*05, *q*50 and *q*95 (CRU data, 1880-2018)

	comb1		comb2		comb3	
q05						
forecast horizon	Forecast	CI	Forecast	CI	Forecast	CI
h=1	2.09	(0.49,3.70)	2.09	(0.48,3.70)	2.15	(0.29,4.01)
h=10	2.43	(0.94,3.92)	2.43	(0.94,3.92)	2.29	(0.80,3.79)
h=25	2.43	(0.89,3.97)	2.43	(0.89,3.97)	2.10	(0.57,3.63)
q50						
forecast horizon	Forecast	CI	Forecast	CI	Forecast	CI
h=1	11.04	(10.22,11.86)	11.04	(10.22,11.87)	11.05	(10.16,11.93)
h=10	11.39	(10.62,12.15)	11.39	(10.62,12.16)	11.09	(10.33,11.85)
h=25	11.67	(10.88,12.45)	11.65	(10.86,12.45)	11.37	(10.60,12.15)
q95						
forecast horizon	Forecast	CI	Forecast	CI	Forecast	CI
h=1	26.76	(26.08,27.44)	26.75	(26.07,27.43)	26.66	(25.90,27.42)
h=10	27.21	(26.60,27.82)	27.21	(26.60,27.82)	27.17	(26.56,27.78)
h=25	27.59	(26.97,28.21)	27.59	(26.97,28.21)	27.72	(27.11,28.32)

Note. comb1: BIC weights; comb2: sample average; comb3: $\beta' s$ from minimizing MSEs, following Bates and Granger (1969-2018).The average values of q05, q50 and q95 for the full sample are, respectively, 0.85, 9.48 and 25.87; in the Paris agreement baseline period (1880-1900) are 0.07, 8.84 and 25.64; in the climatologists reference period (1986-2005) are 1.45, 10.05 and

Long-term temperature Density forecast using the last 70 years

Table 24: Long-term temperature density forecast in the last 70 years (CRU data, 1949-2018) (ACCELERATION HYPOTHESIS)

Models/horizon	h=1			h=25		
	q5	q50	q95	q5	q50	q95
mean	1.19	9.82	26.13	1.19	9.82	26.13
linear-trend	2.03	10.62	26.82	2.60	11.16	27.29
pol-trend	2.78	11.14	26.76	5.35	13.07	25.56
pol-trend-av-sl	2.78	11.14	26.80	3.35	11.68	27.09
pol-trend-log	2.08	10.86	26.97	2.60	11.65	27.60
pol-trend-arp	2.72	11.11	26.65	4.11	12.33	21.03
pol-trend-arp-av-sl	2.72	11.11	26.73	3.32	11.74	24.88
arp	1.67	10.83	26.65	1.24	10.58	27.10
rw	2.70	11.36	26.50	2.70	11.36	26.50
rwd	2.71	11.38	26.51	3.05	11.80	26.77
ima	2.37	11.04	26.77	2.70	11.46	27.03
arfima	1.92	10.87	26.55	1.33	10.19	26.32
arp20	3.13	10.75	26.92	4.34	10.78	22.02
combined	2.36	10.94	26.69	2.92	11.42	25.83
combined*	2.40	10.99	26.70	2.83	11.36	26.14
combined**	2.44	10.99	26.70	2.87	11.37	26.36

Note. The average values of q05, q50 and q95 for the full sample are, respectively, 0.85, 9.48 and 25.87; in the Paris agreement baseline period (1880-1900) are 0.07, 8.84 and 25.64. “Combination” weights the models using the BICs obtained in-sample; “combination*” removes two extreme values and “combination**” four extreme values. In these last cases BIC-weights are properly

Remarks

Some Remarks:

- Global Warming Forecast: q05, q50 and q95 increase with *h*.
- Asymmetric Global Warming Forecast: q05 and q50 increase more than q95 with *h*.
- The Density Forecast shrinks with *h* (Range gets smaller).
- Many models produce a clear acceleration in q05, q50 and q95 forecast.
- Some models present a higher acceleration in q05 implying a shrinking process in the *Range* measure.

Outline

Conclusions and Future research

- Standard climate analysis focus on the Mean temperature. This is ok BUT offers an incomplete picture.
- In this research by using the methodology (realized quantiles) introduced in Gadea and Gonzalo (Joe 2020) we are able to produce wide angle pictures of the future temperature (with standard time series econometrics tools):
  - q05, q50, q95 will increase in the future.
  - q05 increases more than q95. This **climate heterogeneity** may demand heterogenous mitigation policies.
  - There is a clear **acceleration** process in q05 and q50.
  - Notice that an increase in q05 has more serious consequences than the same increase in q95.
  - It seems that the $+2^{o}C$ upperbound of the Paris agreement will be overpassed during the next 25 years, AND we may be reaching some dangerous "tipping" points.
  - Our long-term Mean forecasts are very similar to the ones obtained by climate models projections under RCP 8.5 scenario (”business as usual”).

Conclusions and Future research

- Future Climate Agreements should consider the whole Temperature Distribution and not only the Average.
- Future Research: Forecast combination (confidence intervals, ...); Causal Analysis (different greenhouse gases), comparison with climate models projections, ..., etc.

Conclusions and Future research

THE FUTURE is not what it used to be

THE FUTURE will be warmer

THE FUTURE looks scary

THANKS for attending this not very optimistic presentation!!!!

GG 2019 Methodology

Throughout this paper, similarly to the assumptions made in Park and Qian (2012) and Chang et al. (2016), we assume that in each period, t, there are sufficient temporal or cross-sectional observations ($N \to \infty$) for these characteristics to be estimated consistently.

Assumption: In each period, t, the stochastic functional process $X = (X_t(\omega), t \in T)$ satisfies certain regularity conditions, such that the state densities, distribution and, therefore, quantiles are estimated consistently.

In the temporal framework, local stationarity (Dahlhaus, 2009) plus some strong mixing conditions (Hansen, 2008) are sufficient to obtain uniform ""strong"" consistency for suitable regular kernel estimators of the state densities. Local stationarity plus some φ-mixing conditions (see Degenhardt et al. 1996) are sufficient for the central limit theorem to hold for smoothed empirical distribution functions and for smoothed sample quantiles for each period or segment, t. For the cross-sectional situation, similar results hold (e.g., Silverman, 1978) if the state distributions are defined as cross-sectional distributions, and if independent and identically distributed observations are available to estimate them for each period (for dependency among N observations, see Bosq (1998, Thm 2.2)). Back

GG 2019 Methodology: Proposition 1

Proposition 1

Let $C_t = I(0)$. In the LS regression

$$C_t = \alpha + \beta t + u_t, \tag{2}$$

the OLS estimator satisfies

$$T^{3/2}\widehat{\beta} = O_p(1) \tag{3}$$

and asymptotically ($T \to \infty$)

$$t_{\beta=0} \text{ is } N(0, 1).$$

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GG 2019 Methodology: Proposition 2

Proposition 2

Let $C_t = h(t) + I(0)$, such that $h(t)$ is an increasing $S(\delta)$ function with $\delta \geq 0$, and the function $g(t) = h(t)t$ is $S(\delta + 1)$. In the LS regression

$$C_t = \alpha + \beta t + u_t, \tag{4}$$

the OLS $\widehat{\beta}$ estimator satisfies

$$T^{(1-\delta)}\widehat{\beta} = O_p(1). \tag{5}$$

In order to analyze the behavior of the t-statistic $t_\beta = 0$, we assume that the function $h(t)^2$ is $S(1 + 2\delta - \gamma)$, with $0 \leq \gamma \leq 1 + \delta$. Then, the t-statistic diverges at the following rates

$$t_{\beta=0} = \begin{cases} O_p(T^{\gamma/2}) \text{ for } 0 \leq \gamma \leq 1 \\ O_p(T^{1/2}) \text{ for } 1 \leq \gamma \leq 1 + \delta. \end{cases} \tag{6}$$

GG 2019 Methodology: Proposition 3

Proposition 3

Let $C_t = \mu + z_t, t = 1, ..., T$, with z_t any of the following three processes: (i) a fractional or long-memory model, with $1/2 < d < 3/2$; (ii) a near-unit-root AR model; or (iii) a local-level model. Furthermore, $T^{-1/2+\delta} \sum_{t=1}^{T} z_t \longrightarrow \sigma\ H(.)$, where "$\delta$" is a model-specific constant and H is a model-specific zero-mean Gaussian process with a given covariance kernel $k(r, s)$. Then, in the LS regression

$$C_t = \alpha + \beta t + u_t,$$

the t-statistic diverges,

$$t_{\beta=0} = O_p(T^{1/2}).$$

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Average slope trend models

The $average - slope$ models are time varying linear trend models derived from their corresponding polynomial trend models in the following way:

$$C_t = \beta_0 + \tau_T t + u_t \tag{7}$$

where

$$\tau_T = \frac{1}{T} \sum_{t=1}^{T} \frac{\partial}{\partial t} C_t \tag{8}$$

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