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Wavelet-based Forecasting: An Empirical Comparison of Different Methods

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Motivation

- Time series forecasts are needed in various situations in business.
- In order to produce unskewed forecasts, it is necessary to identify and filter seasonal or other deterministic effects (Wold‘s theorem).This is commonly done using methods like (S)ARIMA or Fourier transform. These are…
  - (+) …simple, comprehensible and easy to implement,
  - (−) …unable to capture seasonalities with dynamic period and/or amplitude.
- An alternative method is required which is able…
  - to identify the temporal character of seasonal oscillations, and
  - to quantify this influence.

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Wavelet Transform
Definition of wavelets

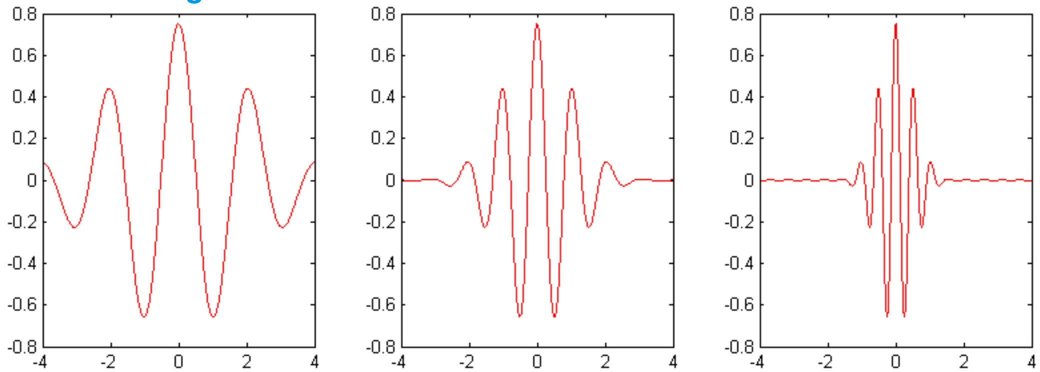
„What if we could use a localized signal instead of a constant sine function?“ Such a signal is called wavelet, if it is a complex-valued function $\Psi(t) \in \mathcal{L}^1(\mathbb{C}) \cap \mathcal{L}^2(\mathbb{C})$ and fulfills

$$C_\Psi = \int_{-\infty}^{\infty} \frac{\left|\widehat{\Psi}(\omega)\right|^2}{|\omega|} d\omega < \infty \,, \|\Psi\| = 1.$$

Thereby, the hat denotes the Fourier transform. Problem: the wavelet has a fixed mean and a constant period → Solution: set $\Psi_{a,b} = \Psi(t/a - b)/\sqrt{a}$.

Example: Morlet wavelet $\Psi = \pi^{-1/4} e^{i\omega_0 t} e^{-t^2/2\sigma^2}$ (see Figure 1).

Figure 1: Morlet Wavelet at three different Scales



Wavelet Transform
Definition of the wavelet transform

The (continuous) wavelet transform of a signal $(X_t)_{\mathrm{t}\in[-\infty,+\infty]}$ is its orthogonal projection on a shifted and scaled wavelet. Using a shift parameter $b \in \mathbb{R}$ and a scaling parameter $a \in \mathbb{R}^+$ the formula reads as follows:

$$\mathrm{WT(a,b)} = \langle X_t, \Psi_{a,b} \rangle = \int_{-\infty}^{\infty} X_t \frac{1}{\sqrt{a}} \Psi^* \left(\frac{t-b}{a} \right) dt.$$

$WT(a,b)$ measures how much of the signal at time b is explained by a wavelet at scaling level a. The higher the coefficient the more similar are signal and wavelet at this specific time.

The inverse transform reads as:

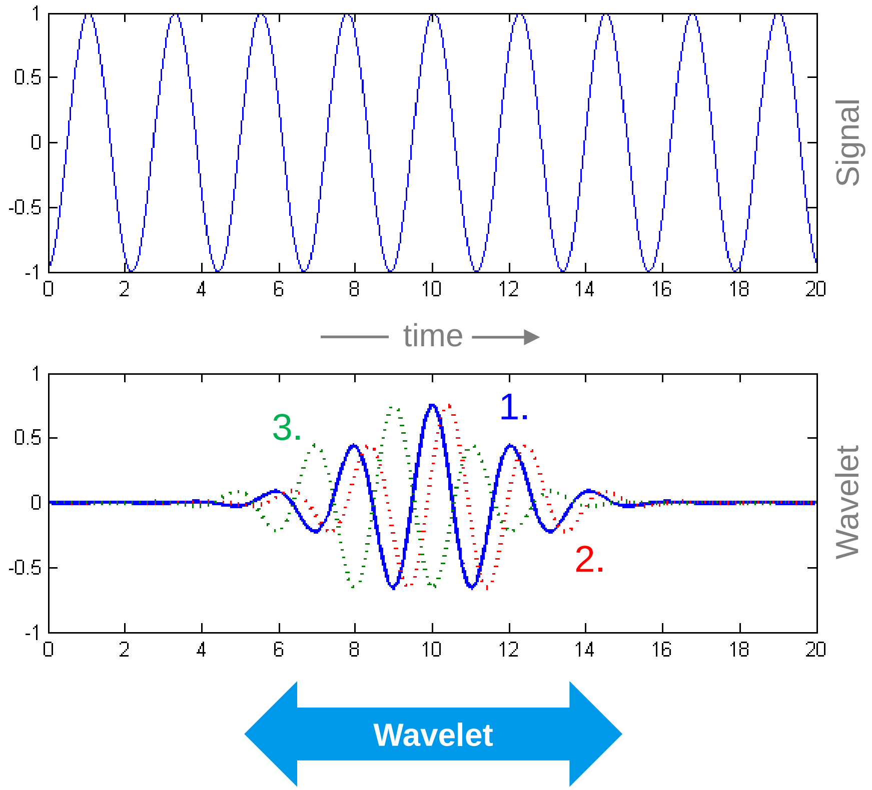
$$X_t = \frac{1}{C_\Psi} \int_0^{\infty} \int_{-\infty}^{\infty} WT(a,b) \frac{1}{a^2 \sqrt{a}} \Psi \left(\frac{t-b}{a} \right) db\, da.$$

For adivce on pracital use, see e.g. Lau & Weng (1995), Torrence & Compo (1998) or Schlueter (2010).

Wavelet Transform

Interpreting the coefficients

Figure 2: Wavelet transform explained



1. Wave peak meets signals peak; signal‘s frequency = wavelet‘s frequency → maximum coefficient $WT(a,b)$.

2. Shifting the wavelet in time → coefficient shrinks

3. Wave peak meets wave trough → minimum coefficient $WT(a,b)$.

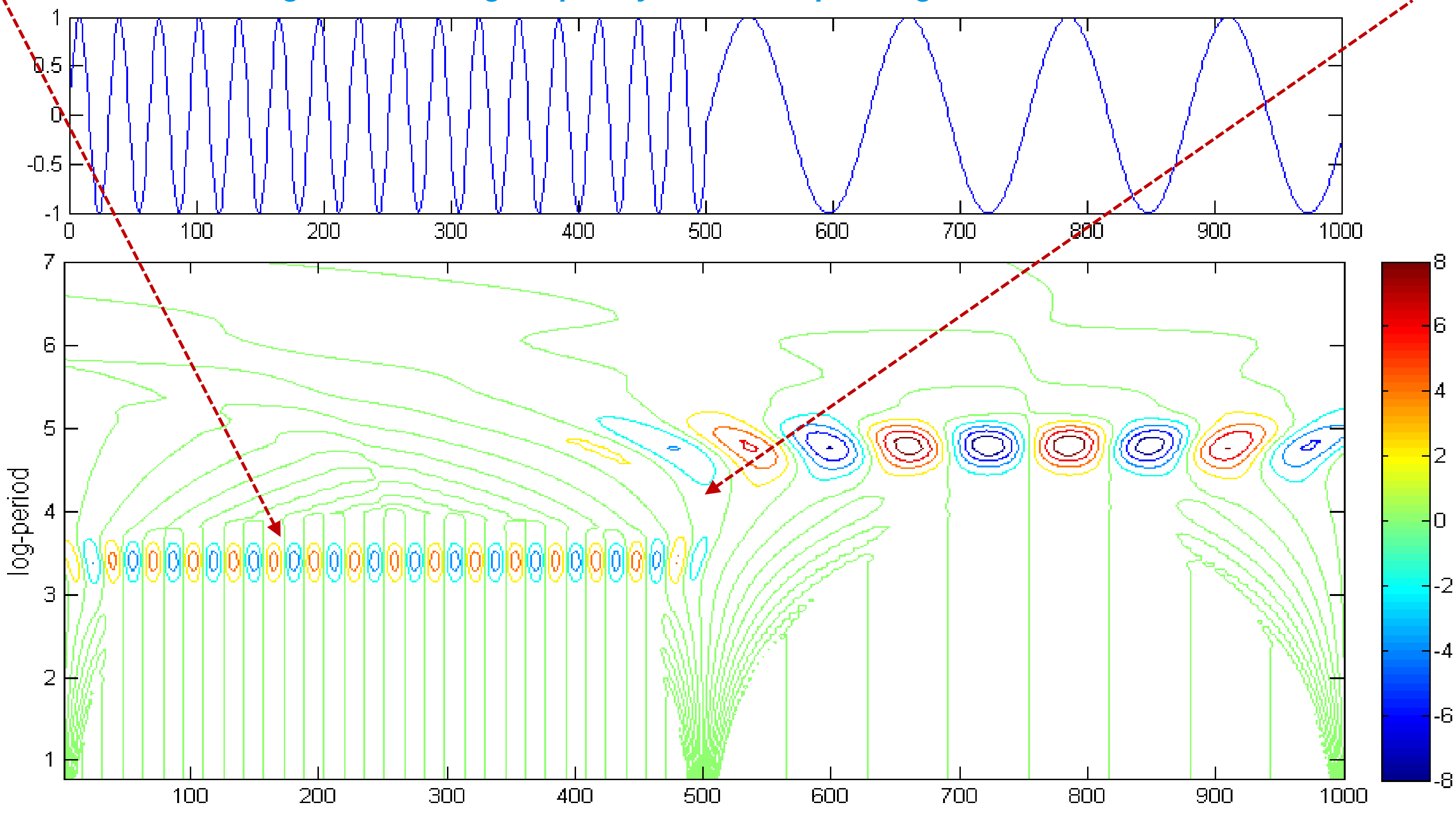
➡ Coefficients oscillate with the wavelet‘s shift in time. Analogously, the coefficients react to changes in the scaling parameter (e.g. stretching).

Wavelet Transform
A motivating example

Coefficients oscillate in both dimensions

Frequencies are seperated

Figure 3: Shifting frequency and corresponding wavelet transform



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Wavelet-Based Forecasting Methods
Denoising

Assumtion: A signal $(X_t)_{t=1,\ldots,T}$ can be split into a deterministic function Y_t and noise ϵ_t.

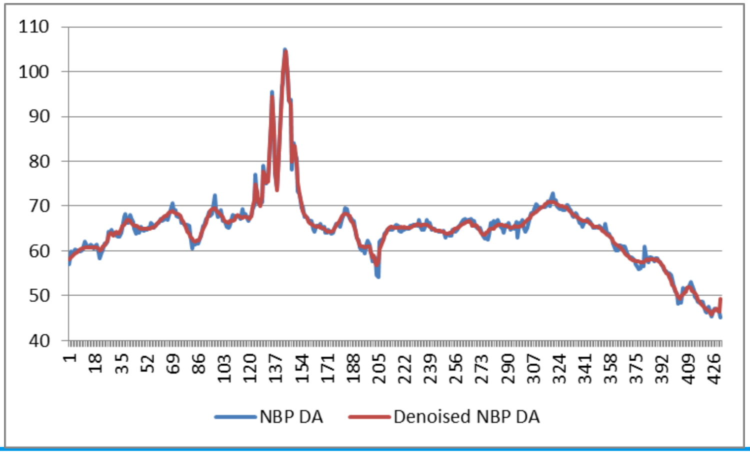
Procedure: Wavelets are used to eliminate/reduce the effect of ϵ_t, which is done as follows (see Donoho & Johnstone, 1994):

1. Apply wavelet transform on X_t and obtain a matrix of wavelet coefficients $WT(a, b)$ with $a \in \{a_0, \ldots, a_n\}$ and $b = 1, \ldots, T$.
2. Now set all coefficients below a certain threshold to zero, i.e.

$$WT'(a,b) = WT(a,b) \cdot 1_{\{|WT(a,b)|>\lambda\}}$$

with $\lambda = \hat{\sigma}\sqrt{2logT}$ and $\hat{\sigma}$ being the wavelet coefficients' standard deviation at scale a_0.

3. Apply the inverse transform to the data.
4. Apply standard forecasting techniques.

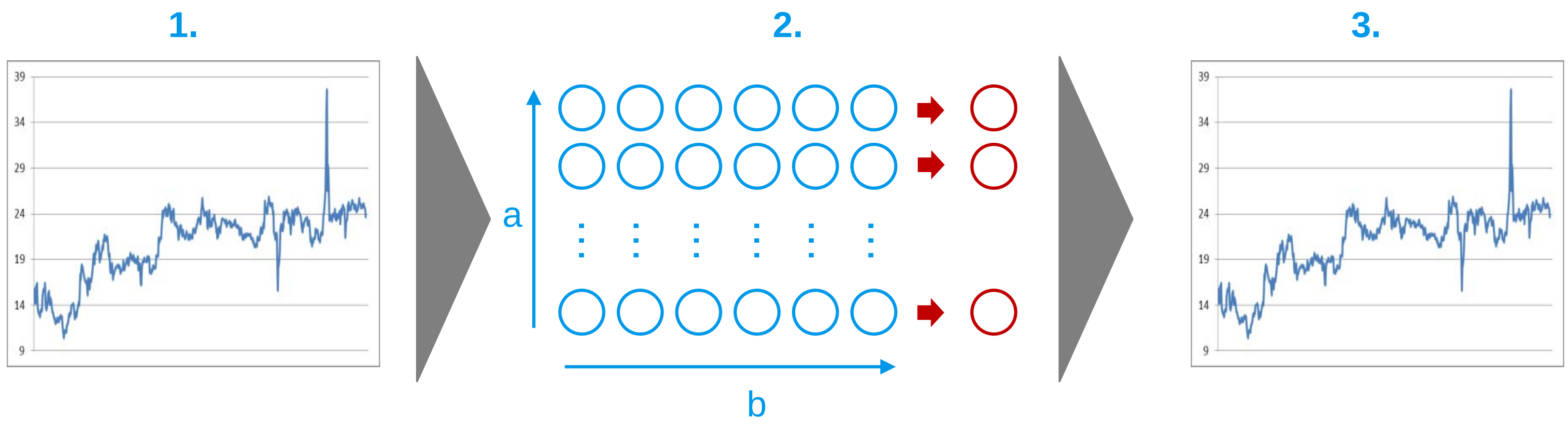


Wavelet-Based Forecasting Methods
Multiscale forecasting

Assumtion: Prices are determined by different traders, each with individual intentions and investment horizons → Wavelets are used to “unbundle“ the influence of these traders.

Procedure:

1. Apply wavelet transform on X_t and obtain a matrix of wavelet coefficients $WT(a, b)$ with $a \in \{a_0, \dots, a_n\}$ and $b = 1, \dots, T$.
2. Standard forecasting techniques are applied to forecast each series of wavelet coefficients.
3. Eventually apply the inverse transform to generate values for $X_t, t > T$, i.e. forecasts.



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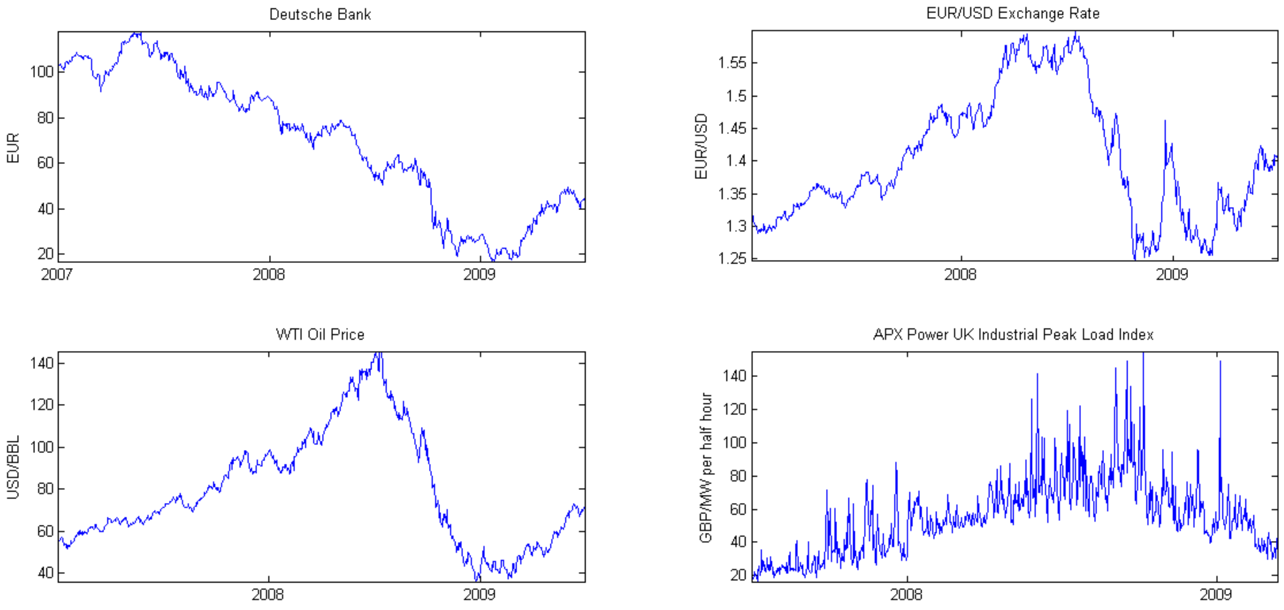
Case Study
The analyzed data sets

Four different time series are analyzed, each having its own characteristics (see Figure 4).

As goodness-of-fit measures we choose the root mean squared error, the mean absolute percentage error and the mean absolute deviation.

Figure 4: The analyzed data sets



Case Study
Results

- For almost all data sets, the use of wavelets does indeed improve the forecasting quality (see Table 1).
- However, the effect heavily depends on the structure of the time series.
- UK power prices are rather short-term with less long-term components → effect of wavelets is small.
- The performance of wavelet-based methods increases with growing forecasting horizon.

Table 1: Performance of different models

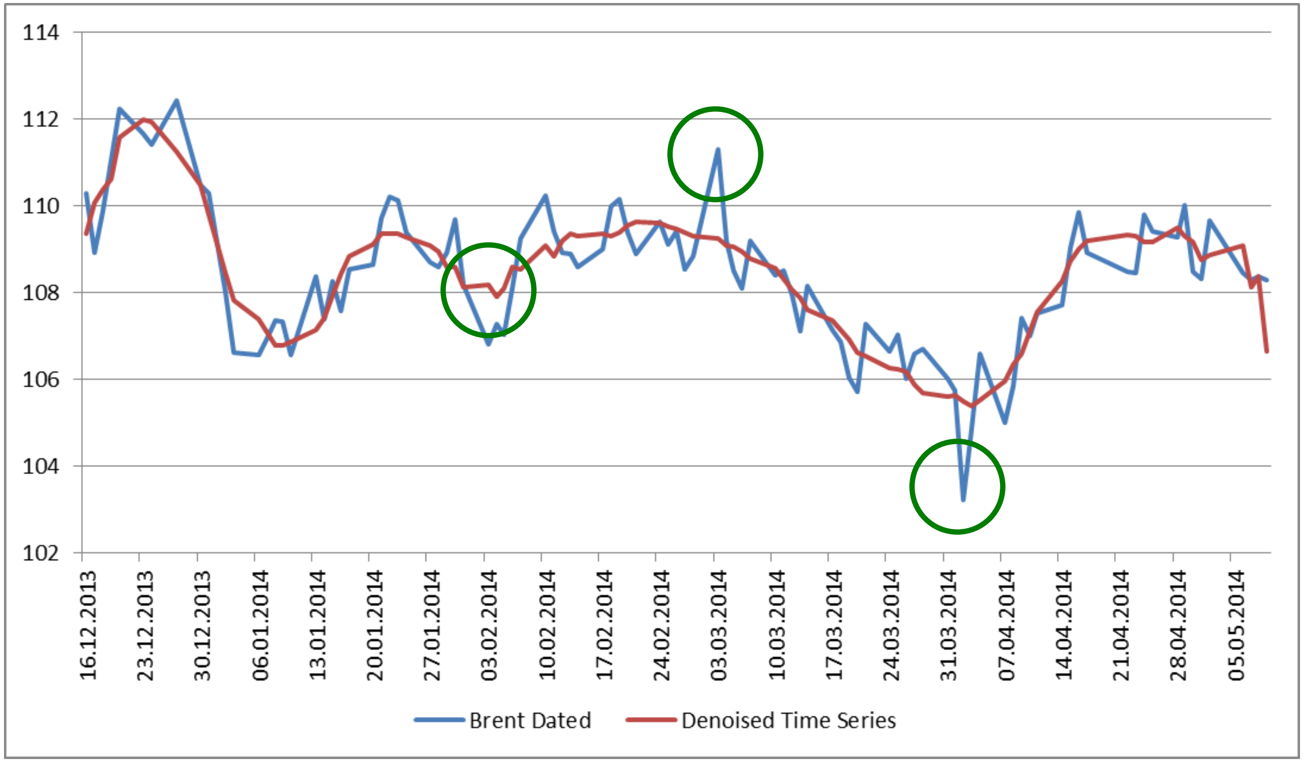
Data Set	Day Ahead	Week Ahead
Deutsche Bank	Multiscale + ARIMA	Multiscale + SARIMA
EUR/USD	Multiscale + ARIMA	Denoising + ARIMA
WTI	Denoising + ARIM A	Multiscale + ARIMA
UK Power Prices	Multiscale + ARIMA	ARIMA

Case Study
Why does denoising work?

Denoising moves the price from the current level to a stable mid-term mean → short-term spikes are ignored.

This is an asset especially for forcasting beyond day ahead.

Figure 5: The analysed data sets



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Conclusion

- Wavelet transform is a tool that helps to identify dynamic seasonal patterns with changing period and amplitude.
- Hence, wavelet transform can be applied to identify dynamic structures in a time series, e.g. to unbundle the effect of traders with different investment strategies.
- This knowledge can be used to improve forecasting quality.
- Results:
  - Performance of wavelet-based forecasting techniques improves with growing forecasting horizon.
  - Performance relies on the existence of long-term structures in a data set.

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