Predicting U.S. Stock Market Return: Evidence from the improved Augmented Regression Method

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Motivation

- Forecasting returns is interesting to both academics and practitioners.
- Extensive literature on the predictability of aggregate stock market returns.
- Lack of consistency in the evidence on stock market predictability over time.
- A host of econometric issues allowing for advancing estimation and testing methods.
- Inconsistencies between in-sample and out-of-sample return predictability results in a number of studies.
- Apply the improved ARM to test the predictability of the S&P 500 equity premium.
Research Questions

- What does the improved ARM say about the predictability of the S&P500 equity premium?
- Does predictability change over time?
- How significant is the effect size over time?
- How do financial predictors perform in in-sample and out-of-sample forecast evaluation tests?
- Do investors derive economic utility by using the improved ARM to forecast the equity premium?
Main Findings

- Evidence of occasional in-sample predictability, however, the effect size is small.
- DY and MA are the most prominent predictors in-sample.
- Weak evidence of out-of-sample predictability.
- Inconsistency between in-sample predictability and out-of-sample predictability.
- Investors do realize occasional utility gains.
Potential Predictors

- Predictive regression:

\[ Y_t = \beta_0 + \beta_1 X_{t-1} + u_t \]
\[ X_t = \rho_0 + \rho_1 X_{t-1} + \nu_t \]


- Data mining and other issues.
Selected Literature

- Stambaugh (1999): \( E(\hat{\beta}_1 - \beta) = \frac{\sigma_{u,v}}{\sigma_v^2} E(\hat{\rho}_1 - \rho) \approx -\frac{\sigma_{u,v}}{\sigma_v^2} \left( \frac{1+3\rho_1}{n} \right) \)

- Lewellen (2004): estimates \( \beta_1 \) and its t-stat. under a conservative assumption that the true autoregressive coefficient \( \rho \approx 1 \). This places an upper bound for the bias in \( \hat{\beta}_1 \). (preserve predictability).

- Recent empirical findings on predictability in the US: Campbell and Thompson (2008); Goyal and Welch (2008); Rapach and Wohar (2006); Neely, Rapach, Tu and Zhou (2014); Cenesizoglu and Timmermann (2012)
Predictive Regression: Augmented Regression Model

- Augmented regression model - Amihud et al. (2010)

\[ Y_t = \beta_0 + \beta_1 X_{t-1} + \cdots + \beta_p X_{t-p} + u_t \]
\[ X_t = \rho_0 + \rho_1 X_{t-1} + \cdots + \rho_p X_{t-p} + v_t \]

- Hypotheses of interest:

\[ H_{01} : \beta_1 = \cdots = \beta_p = 0 \]
\[ H_{02} : \beta_1 + \cdots + \beta_p = 0 \]

- Amihud and Hurvish (2004) and Amihud et al. (2008, 2010) allow for linear dependence between the error terms the predictive regression as \( u_t = \phi v_t + \epsilon_t \) and run the regression:

\[ Y_t = \beta_0 + \beta_1 X_{t-1} + \cdots + \beta_p X_{t-p} + \phi v_{t}^{bc} + \epsilon_t \]

where \( v_{t}^{bc} = X_t - (\widehat{\rho}_0^{bc} + \widehat{\rho}_1^{bc} X_{t-1} + \cdots + \widehat{\rho}_p^{bc} X_{t-p}) \)

- The bias correction applies Shaman and Stine (1989) bias formulae. The regression is estimated to obtain \( \beta_i^{bc} \) and the corresponding covariance matrix for hypothesis inference.
Improved the accuracy of the bias-corrected estimators.

Faster convergence of the bias correction relative to the ARM.

Correcting the bias in estimators may render them non-stationary. When this is the case, a stationarity-correction is applied (Kilian, 1998).

Derive a matrix formula for bias corrections which facilitates the implementation of higher order models.

Applied to multiple predictors.

Monte Carlo evidence shows that parameters estimates using the improved ARM are more accurate than the ARM in small samples.
Regress $Y_t$ on $(1, X, v^a)$ where

$$v^a = X_t - \hat{\rho}_0 + \hat{\rho}_1 X_{t-1} - \cdots - \hat{\rho}_p X_{t-p}$$

Bias correction for autoregressive estimators $\rho$

Matrix formulae for bias-correction and covariance matrix estimation

Kim (2004) shows that $E(\hat{\gamma}^a - \gamma) = O(n^{-2})$

Bias correction converges at a rate of $n^{-2}$.

Stationarity

Use Killian’s (1998) stationarity correction for a possible non-stationary $\rho$ over lags $p$.

Extended to lag order $p$

- In-sample $F$-test
- Out-of-sample forecasting for any $h > 0$. 

Predicting U.S. Stock Market Return

Jurdi and Kim (La Trobe University)
OOS Forecast Evaluation

\[ \text{Theil} - U = \frac{\sqrt{\sum (y_t - \hat{y}_t^f)^2}}{\sqrt{y_t^2 + \hat{y}_t^f}} \]

Economic Utility

- At the end of month \( t \), an investor allocates \( w_t \) to equities during month \( t + 1 \) to equities using iARM or HA as

\[ w_t = \left( \frac{1}{\gamma} \right) \left( \frac{\hat{y}_{t+1}}{\hat{o}_{t+1}^2} \right) \]

and the rest to a risk free asset.

- Following Campbell and Thompson (2008), allow \( 0 \leq w_t \leq 1.5 \).

- \( CER_p = \hat{u}_p - \frac{1}{2} \gamma \hat{o}_p^2 \) and \( CER \text{ gain } = (CER_{iARM} - CER_{HA}) \)
Data and Estimation

- Monthly data from January 1926 to December 2012 for the S&P 500 and predictors obtained from Amit Goyal’s website.
- Daily data obtained from Bloomberg to construct moving average predictors.
- Predictors: DY; PE; BM; TERM spread*; DEF spread*; inflation*; risk free interest rate*; price moving average
- Estimation:
  - Moving sub-sample window of 10 years (120 observations) Hsu and Kuan (2005).
  - Predictor lag order determined by the Akaike information criterion
  - Out-of-sample forecasts $h = 1, \ldots, 12$
  - Statistical and economic forecast evaluation.
Results - DY

DY has no predictability

Effect Size Estimates and 95% Confidence Intervals: DY

Thall's U: DY
Results - PE

PE has no predictability

Effect Size Estimates and 95% Confidence Intervals: PE

Thaler’s U: PE
Results - BM

BM has no predictability

Effect Size Estimates and 95% Confidence Intervals: BM

Theil's U: BM
Results - MA (1-50)

- MA has no predictability
- Effect Size Estimates and 95% Confidence Intervals MA
- The 95% MA
Results - Economic Gains I

Utility Gains - DY

Utility Gain - PE
Multivariate iARM

DY has no predictability

PE has no predictability

BM has no predictability
**Conclusion**

- Statistically significant in-sample predictability occurs only temporarily and sporadically.
- Weak out-of-sample predictability.
- Inconsistencies between in-sample and out-of-sample predictability.
- Multivariate predictive models results confirm the findings of univariate models.
- Occasionally investors realize positive economic gains.
- Results from a multipredictor excercise show qualitatively similar results.